1. Consider the surface \( S \) given by

\[ x^3 + xy^3 + yz^3 + z = 40 \]

and the point \( P(3, 1, 2) \) on \( S \).

a) Find an equation for the tangent plane to \( S \) at \( P \).

b) Find parametric equations for the normal line to \( S \) at \( P \).
2. Given \( f(x, y) = xe^{xy} \):

a) Find the gradient vector field \( \nabla f \).

b) At the point \( (x, y) = (2, 1) \), find the rate of change of \( f \) in the direction of the vector \( \vec{i} - \vec{j} \).

c) Find a unit vector which points in the direction you should follow if, starting at \( (2, 1) \), you want to achieve the most rapid increase possible for \( f(x, y) \).
(18) 3. Given \( f(x, y) = x^2y^3 + 2xy - 3y \):

a) Calculate all the first and second partial derivatives of \( f(x, y) \).

b) Find all the points where the tangent plane to the graph of \( f(x, y) \) is horizontal.

c) Use the 2nd derivative test to classify all those critical points of \( f(x, y) \) to which the test is applicable.
4. Find the absolute maximum and minimum values for \( z = xy^2 \) in the disk \( x^2 + y^2 \leq 9 \). (For 3 pts. extra credit, explain very briefly why it is that \( z \) is guaranteed to have absolute extrema in the given region.)
5. Use the method of Lagrange multipliers to find the largest value and the smallest value of $f(x, y, z) = 2x + 4y + z$ on the ellipsoid $2x^2 + y^2 + z^2 = 19$. 
6. The temperature at a point \((x, y)\) in the plane is given by \(T(x, y) = x + y^2 - 3\). Find a parametrization \(\vec{r}(t)\) for the path of a heat-seeking particle which, at time \(t = 0\), lies at the point \((1, 2)\).

7. Given \(z = f(x, y)\) where \(x = s + 2t\) and \(y = 3s + 2t\); suppose you know that at the point \((x, y) = (1, 1)\) you have \(\frac{\partial z}{\partial x} = 3\) and \(\frac{\partial z}{\partial y} = 4\). Find \(\frac{\partial z}{\partial s}\) and \(\frac{\partial z}{\partial t}\) at the given point.