TO RECEIVE CREDIT YOU MUST SHOW YOUR WORK.

(20) 1) Find the equation of the tangent plane to the given surface at the specified point.

   a) To $x^3z + y^3x + z^3 = 38$ at $(x, y, z) = (1, 2, 3)$.

   b) To $z = xe^{xy}$ at $(x, y, z) = (1, 2, e^2)$. 
(20) 2) Let $f(x, y) = \sqrt{y^2 + 2x^2}$.

a) Find the gradient vector field of $f(x, y)$,
\[ \nabla f = \]

b) Find the directional derivative of $f(x, y)$ at $(x, y) = (2, 1)$ in the direction of the vector $\vec{a} = -3\vec{i} + 4\vec{j}$.

c) Find a vector which points in the direction which gives the largest directional derivative for $f(x, y)$ at $(x, y) = (2, 1)$. Find the value of this largest directional derivative at $(2, 1)$. 
3) Suppose that \( z \) is a function of \( x \) and \( y \), \( z = f(x, y) \), and \( x \) and \( y \) are functions of \( u \) and \( v \) according to \( x = \frac{u}{v} \), \( y = u^2v \). Suppose that at \((x, y) = (2, 4)\) you know that \( \frac{\partial f}{\partial x}(2, 4) = 5 \) and \( \frac{\partial f}{\partial y}(2, 4) = 2 \). Use the chain rule to find \( \frac{\partial z}{\partial u} \) and \( \frac{\partial z}{\partial v} \) at the point \((x, y) = (2, 4)\).
(20) 4) Use the method of Lagrange multipliers to find the largest value and the smallest value for $f(x, y, z) = x^2 + 4y + 4z$ on the ellipsoid $x^2 + 2y^2 + z^2 = 24$. 
(20) 5. Find and classify the critical points of

\[ f(x, y) = x^2 + y^3 - 6xy + 3x + 6y. \]