1. Find the equation for the tangent plane to the given surface at the specified point.

   a) \( x^3y + yz^2 + xz = 23 \) at \((1, 2, 3)\)

   b) the graph of \( f(x, y) = xe^{xy} \) at \((2, 3, 2e^6)\)
2. Given that \( z = f(x, y) \) and \( x = u - 2v, \ y = u^3 + v^3 \). Suppose that at the point \((x, y) = (0, 9)\) you know that \( \frac{\partial f}{\partial x} (0, 9) = 4 \) and \( \frac{\partial f}{\partial y} (0, 9) = 2 \). Use the Chain Rule to calculate \( \frac{\partial z}{\partial u} \) and \( \frac{\partial z}{\partial v} \) when \( x = 0 \) and \( y = 9 \).
3. Let \( f(x, y) = y \sqrt{x^2 + y^2} \).

a) Find the gradient vector field of \( f(x, y) \).
\[
\nabla f =
\]

b) Find the directional derivative of \( f(x, y) \) at \((x, y) = (3, 4)\) in the direction of the vector \( \vec{a} = -\vec{i} + 2\vec{j} \).

c) Find the largest directional derivative of \( f(x, y) \) at \((x, y) = (3, 4)\). In which direction is it obtained?
(20) 4. Use the method of Lagrange multipliers to find the largest value and the smallest value of \( f(x, y) = x^2 + y^2 \) on the curve \( x^4 + 16y^4 = 16 \).
20) 5. Find and classify the critical points of \( f(x, y) = x^2y - \frac{2}{3}x^3 + 3y - 2y^2 \).