1. Find the tangent plane to the given surface at the specified point.

   a) To the graph of $f(x, y) = xy$ at the point $(x, y, z) = (2, 3, 6)$.

   b) To $x^3y + y^3z + xz^3 = 53$ at $(x, y, z) = (1, 2, 3)$. 

2. Let \( f(x, y) = y\sqrt{x^2 + y^2} \).

a) Find the gradient vector field of \( f \).

\[ \nabla f = \]

b) Find the directional derivative of \( f \) at \((x, y) = (3, 4)\) in the direction of the vector \( \mathbf{v} = 2\mathbf{i} - \mathbf{j} \).

c) Find a vector which points in the direction that gives the largest directional derivative for \( f \) at \((3, 4)\). What is the value of this largest directional derivative?
(20) 3. Find and classify the critical points of \( f(x, y) = 4xy - y^4 - x^2 \).
(15) 4. Use the method of Lagrange multipliers to find the largest value and
the smallest value of $f(x, y) = x^2 + 2x - y^2$ on the circle $x^2 + y^2 = 16$. 
5. Use a double integral to calculate the volume of the 3-D region which is under \( z = 1 + xy \) and above the region in the first quadrant of the \( xy \)-plane which is bounded by \( y = 2x, x = 0 \) and \( y = 4 \).
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(10) 6. Evaluate \[ \int_{0}^{2} \int_{y^2}^{4} \sqrt{1 + x \sqrt{x}} \, dx \, dy \] by first reversing the order of integration.
7. Use a double integral to calculate the volume of the 3-D region which is enclosed by the paraboloids $z = 8 - x^2 - y^2$ and $z = x^2 + y^2$. Use polar coordinates to evaluate the double integral.