1) Find the volume of the 3D-region which is under $z = 1 + x^2$ and above the bounded region in the first quadrant of the $xy$-plane that is enclosed by the lines $y = x$, $y = 3x$ and $x = 2$. 
(15) 2) Use the method of Lagrange multipliers to find the largest value and the smallest value of \( f(x, y) = x + y^2 \) on the ellipse \( x^2 + 2y^2 = 16 \).
(20) 3) Find and classify the critical points of

\[ f(x, y) = 12xy - 3x^2 + y^3 - 15y^2 - 9y. \]
(15) 4) Given that $z$ is a function of $x$ and $y$, $z = f(x, y)$, and that

$$x = s^2 + t, \quad y = \frac{2s}{t}$$

with $s > 0$ and $t > 0$.

Suppose that at $(x, y) = (3, 1)$ we know that $\frac{\partial f}{\partial x}(3, 1) = 2$ and $\frac{\partial f}{\partial y}(3, 1) = 6$. Use the chain rule to find the values of $\frac{\partial z}{\partial s}$ and $\frac{\partial z}{\partial t}$ when $x = 3$ and $y = 1$. 
5) Let \( f(x, y) = 6x \sqrt{x + y^2} \).

a) Find the gradient vector field of \( f \), \( \nabla f = \)

b) Find the directional derivative of \( f(x, y) \) at \( (x, y) = (5, 2) \) in the direction of the vector \( \vec{v} = 2\vec{i} - 3\vec{j} \).

c) What is the value of the largest directional derivative of \( f(x, y) \) at \( (5, 2) \)? Find a unit vector which points in the direction which gives this largest directional derivative.
(20) 6) Find the equation of the tangent plane to each of the surfaces at the specified point.

a) to $z = ye^{xy}$ at $(1, 2, 2e^2)$

b) to $xz^3 + zy^3 + yx^3 = 53$ at $(1, 2, 3)$