(15) 1) Find the equation of the tangent plane to the given surface at the specified point.

a) \( x^3 + y^3 + xyz^3 = 41 \) at \( (3, 2, 1) \)

b) \( z = 2x\sqrt{x + y^2} \) at \( (5, 2, 30) \)
(20) 2) Given \( f(x, y) = ye^{xy} \)

a) Find the gradient vector field of \( f(x, y) \).

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\nabla f = 
\]

b) At the point \((x, y) = (-1, 2)\) find the instantaneous rate of change of \( f(x, y) \) per unit distance in the direction of the vector \( \vec{v} = \vec{i} + 6\vec{j} \).

c) Find a unit vector which points in the direction you should follow, if starting at \((x, y) = (-1, 2)\), you want to achieve the most rapid increase possible for \( f(x, y) \).
3) Let $f(x,y) = 4xy - xy^2 - x^3$. Find and classify the critical points of $f(x,y)$. 
(10) 4) Use the method of Lagrange multipliers to find the maximum value and the minimum value of \( f(x, y, z) = 2x + 3y + z \) subject to the two constraints \( x^2 + y^2 = 20 \) and \( x + y + z = 0 \).
(15) 5) Find the volume of the 3-dimensional region which is under the plane $z = 1 + 2y$ and above the bounded region of the $xy$-plane which is enclosed by $y = x^2$ and $y = 4$. 
(15) 6) Use a double integral in polar coordinates to calculate the volume of the 3-dimensional region enclosed by the surfaces \( z = x^2 + y^2 \) and \( z = 6 - \sqrt{x^2 + y^2} \).