(20pts) 1. Classify the following as either scalar or vector by circling the appropriate letter: s — scalar, v — vector.

a. acceleration (s v)  
   f. dot product (s v)  
b. antigradient (s v)  
g. gradient (s v)  
c. cross product (s v)  
h. speed (s v)  
d. curvature (s v)  
i. torsion (s v)  
e. directional derivative (s v)  
j. velocity (s v)  

(20pts) 2. Use the chain rule to find \( \frac{\partial w}{\partial x} \), where \( w = u^2 e^v \) and \( u = \frac{x}{y} \), \( v = y \ln x \). You need not simplify. However, no \( u \) and \( v \) should appear in your answer.
(20pts) 3. Show that the tangent plane to the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ at $(x_0, y_0, z_0)$ is
\[
\frac{x_0x}{a^2} + \frac{y_0y}{b^2} + \frac{z_0z}{c^2} = 1.
\]
(Note that the point $(x_0, y_0, z_0)$ is on the ellipsoid.)

(20pts) 4. Let $f(x, y) = 4y - x^2 - y^2$. Find a unit vector which produces the smallest directional derivative for $f(x, y)$ at $(1, 1)$. What is the value of this smallest directional derivative?
(20pts) 5. Locate and identify all the local extrema and saddle points of the function \( f(x, y) = 4xy - x^4 - y^4 \).

(20pts) 6. Use a triple integral in spherical coordinates to find the volume of “ice cream cone” above the \( xy \) plane, inside the cone \( x^2 + y^2 = 3z^2 \) and capped by the sphere \( x^2 + y^2 + z^2 = 1 \).
(20pts) 7. Let $\vec{F}(x, y) = (y - x, x^2 y)$. $C$ be a curve along $y = x^2$ from $(0, 0)$ to $(1, 1)$.

(a) Show that $\vec{F}$ is not a gradient field.

(b) Compute $\int_C \vec{F}(\vec{x}) \cdot d\vec{x}$.

(20pts) 8. Let $\vec{F}(x, y) = (\cos y + y \cos x, \sin x - x \sin y)$, $C$ be the curve $y = x^2 / \pi$ from $(\pi, \pi)$ to $(0, 0)$.

(a) Find a scalar field $f(x, y)$ whose gradient is $\vec{F}(x, y)$, i.e., the anti-gradient of $\vec{F}$. 
8. (b) Use the Fundamental Theorem of Calculus for Line Integrals to evaluate: \[ \int_C \vec{F}(\vec{x}) \cdot d\vec{x}. \]

(20pts) 9. Use Green’s Theorem to compute \[ \int_C 2x^2ydx + (x^3 - y)dy, \] where \( C \) is the boundary of the part of the first quadrant cut off by the line \( x + y = 1 \).
10. Let $D$ be any standard region with boundary $C$ (oriented counterclockwise).

(a) Show that the area of $D$ is $A = \frac{1}{2} \int_C x \, dy - y \, dx$. (Hint: Apply the Green’s Theorem to the line integral.)

(b) The area of $D$ can also be computed by $A = -\int_C y \, dx$. Use this formula to find the area bounded by $x$ axis and the curve $y = \sin x$, $0 \leq x \leq \pi$. 