(20) 1. An object is moving in 3-space along the parametric curve

\[ \vec{r}(t) = (t + 1, \sin t, \cos t) \]

Find, as functions of \( t \):

(a) velocity vector \( \vec{v} = \)

(b) acceleration vector \( \vec{a} = \)

(c) speed \( \frac{ds}{dt} = \)

(d) tangential component of acceleration \( a_{\vec{N}} = \)

(e) normal component of acceleration \( a_{\vec{N}} = \)

(f) curvature \( K = \)

(g) the cosine of the angle between \( \vec{v} \) and \( \vec{a} = \)
2. An object is moving in the plane along the curve $y = \sin x$, in the direction of increasing $x$. Its speed is constant at 2 ft/sec.

(a) Find $a_T$ and $a_N$ when the object is at $(x, \sin x)$.

(b) Find the velocity vector and the acceleration vector when the object is at the point $\left(\frac{\pi}{2}, 1\right)$. 
(10) 3. Consider the function $f(x, y, z) = xyz$.

(a) Find the directional derivative of $f$ at $(1, 2, -1)$, in the direction towards the origin $(0, 0, 0)$.

(b) Find the unit vector which points in the direction of the greatest increase in $f$. 
(12) 4. Find an equation for the tangent plane to the given surface $S$ at the given point $P$.

(a) $S$: ellipsoid $2x^2 + y^2 + z^2 = 4$

$P$: $(1, 1, -1)$

(b) $S$: the torus given parametrically by

$$\vec{r}(u, v) = \langle (2 + \cos v) \cos u, (2 + \cos v) \sin u, \sin v \rangle$$

$P$: the point given by $u = v = \pi/2$
5. Find and classify the critical points for the function

\[ f(x, y) = x^3 - 3xy + y^3 + 1. \]
6. Use the method of Lagrange Multipliers to find the largest and smallest values for $4xy$ on the surface $x^4 + y^4 = 32$. 
(18) 7. Given the mass distribution function $\rho(x, y, z) = z$, calculate the mass of the 3D-region under the paraboloid $z = 4 - x^2 - y^2$, above the $xy$-plane, and inside the cylinder $x^2 + y^2 = 1$. 
(15) 8. Find the work done by the force field $\vec{F}(x, y, z) = \langle x, xy \rangle$ in moving an object from $(1, 1)$ to $(2, \frac{1}{2})$ along the curve $y = 1/x$. 
(15) 9. (a) Show that the field
\[
\vec{F}(x, y, z) = (y \sin z + 1, x \sin z, xy \cos z + 1)
\]
is conservative, by finding a potential function for \( \vec{F} \).

(b) Find the work done by the field \( \vec{F} \) from part (a), in moving a particle from \((1, 0, 0)\) to \((-1, 0, \pi)\) along the spiral \( \vec{r}(t) = (\cos t, \sin t, t) \).
10. Use Green’s Theorem to evaluate the line integral

\[ \int_C (x^2 + y^2)dx + 2xy
dy, \]

where \( C \) is the boundary of the triangle with vertices (0, 0), (1, 2), (0, 2), oriented counterclockwise.
11. Let $S$ be the parametrized surface

$$
\vec{r}(u, v) = \langle u \cos v, u \sin v, v \rangle \text{ (spiral ramp) with } 0 \leq u \leq 1, \quad 0 \leq v \leq 2\pi.
$$

Evaluate the surface integral $\int \int_S u \, dS$. 
12. Use the divergence theorem to find the outward flux of the vector field
\( \vec{F}(x, y, z) = \langle 0, 0, z^2 \rangle \) across the hemisphere 
\( z = \sqrt{1 - x^2 - y^2} \).
13. Let $S$ be the sphere $x^2 + y^2 + z^2 = a^2$, with outward-pointing normal orientation, and let $C$ be the boundary of the 1st-octant portion of $S$ (oriented in the standard way with respect to the orientation of $S$). Use Stokes’s theorem to find the work done by the field

$$\vec{F}(x, y, z) = (-y, x, z)$$

in moving a particle once around $C$, in the direction given by $C$’s orientation. 

[Hint: The total area of $S$ is $4\pi a^2$. This should simplify things.]