1) The points $P(1, 0, 2), Q(2, 1, 2)$ and $R(0, -2, 4)$ are vertices of a triangle in 3-space.

a) Find the angle, in degrees, at the vertex $P$.

b) Find the area of the triangle.
(20) 2. Find the equation for each of the following planes.

a) The plane containing the points \( P, Q \) and \( R \) of problem 1.

b) The tangent plane to the surface \( x^2 + z^4 = 5y^2 + 5z^2 \) at the point \((3, 1, 2)\).
3) Let \( f(x, y, z) = xz^2 + yx^2 + zy^2 \).

a) Find the directional derivative of \( f(x, y, z) \) at \((x, y, z) = (1, 2, -1)\) in the direction of the vector \( \vec{a} = \vec{i} + 5\vec{j} + 3\vec{k} \).

b) Find the value of the largest directional derivative of \( f(x, y, z) \) at \((1, 2, -1)\). In which direction does it occur?
(20) 4) An object is moving in 3-space in such a way that its acceleration vector as a function of the time $t$ is

$$\mathbf{a} = (\cos t + \sin t) \mathbf{i} + (\cos t - \sin t) \mathbf{j}. $$

Suppose that at time $t = 0$ its velocity vector is $\mathbf{v}(0) = -\mathbf{i} + \mathbf{j} + 2k$. Find, as functions of $t$, $a_T$, $a_N$, curvature, $\mathbf{T}$ and $\mathbf{N}$. 
(20) 5) Let \( f(x, y) = -x + xy - (e^y)(x^{-1}) \). Find and classify the critical points of \( f(x, y) \) in the region \( x \neq 0 \).
(15) 6) Use the method of LAGRANGE MULTIPLIERS to find the smallest value of \( F(x, y, z) = x^2 + y^2 + z^4 \) on the surface \( xyz = 64 \) for \( x > 0, y > 0 \) and \( z > 0 \).
(10) 7) Calculate the volume of the 3D-region in the first octant which is enclosed by the surfaces $y = x, y = 2, x = 0, z = 2y$ and $z = 6$. 
(15) 8) The force field \( \vec{F} = xy \vec{i} - x \vec{j} \) acts on an object as it moves in the plane. Find the work done by \( \vec{F} \) as the object moves from \((2, 0)\), to \((-2, 0)\) along the upper half of the circle \( x^2 + y^2 = 4 \).
(10) 9) Show that the force field $\vec{F} = (2x + y^{-1}) \hat{i} + (2y - xy^{-1}) \hat{j}$ is conservative in the region $y > 0$ by finding a potential function for it. Now use this potential function to calculate the work done by $\vec{F}$ as it acts on an object which moves from $(-1, 1)$ to $(4, 2)$ along any curve in the upper half plane.
Use Green’s Theorem to evaluate the line integral
\[
\int_C (3y^2 + 3x^2 - y^3)dx + (6xy + x^3)dy
\]
where \( C \) is the curve consisting of the \( x \)-axis from \((-2, 0)\) to \((2, 0)\) followed by the upper half of the circle \( x^2 + y^2 = 4 \) from \((2, 0)\) to \((-2, 0)\).
11) Let $T$ be the 3D-region which is inside the sphere $x^2 + y^2 + z^2 = 4$ and above the $xy$-plane, and $S$ be its boundary. If $\vec{F} = xz \vec{i} + yz \vec{j} + z \vec{k}$, use the divergence theorem to find the value of the outward flux integral of $\vec{F}$ across $S$. 
(20) 12) Let $S$ be that part of $z = x^2 + y^2$ which is in the first octant and under the plane $z = 4$. Suppose that $\vec{F} = y\vec{i} - x\vec{j} + y^2\vec{k}$.

a) Calculate $\text{curl} \vec{F}$.

b) Let $C$ be the positively oriented boundary of $S$ where $S$ is given the upward pointing normal. Use Stokes’ Theorem to find the value of the line integral $\int_C \vec{F} \times \vec{T} \, ds$. 