1) A particle is moving in 3-space. At a certain instant \( t = t_0 \) seconds we know that the position vector is \( \vec{r}(t_0) = 2\vec{i} - \vec{j} + \vec{k} \), the velocity vector is \( \vec{v}(t_0) = \vec{i} + \vec{j} + 3\vec{k} \) and the acceleration vector is \( \vec{a}(t_0) = \vec{i} + 2\vec{j} - 2\vec{k} \). **At this instant** \( t = t_0 \) determine

a) the point at which the particle is located;

b) the speed of the particle;

c) the unit tangent vector;

d) the tangential component of acceleration;

e) the normal component of acceleration;

f) the curvature.

g) Is the object speeding up or slowing down? Why?
an object is moving in 3-space in such a way that its acceleration vector as a function of time is $\vec{a} = \vec{j} + (\cos t)\vec{k}$. Suppose that at time $t = 0$ you know that its velocity vector is $\vec{v}(0) = \vec{i}$ and its position vector is $\vec{r}(0) = \vec{k}$.

a) Find the velocity vector and the position vector as functions of time $t$.

b) Give the parametric equations for the motion.

c) Find the curvature at time $t = 2\pi$. 
(15) 3) A quantity $Q$ depends upon $x$ and $y$ according to $Q = \frac{e^{xy}}{x^2 + y^2}$. Both $x$ and $y$ are changing with time $t$ and suppose at a certain instant we know that $x = 1$, $y = 2$, $\frac{dx}{dt} = -2$ and $\frac{dy}{dt} = 6$. Use the chain rule to find $\frac{dQ}{dt}$ at this instant.
(15) 4) Use the method of Lagrange multipliers to find the maximum value and the minimum value of $f(x, y) = y^2 - 2y - x^2$ on the circle $x^2 + y^2 = 4$. 
(20) 5) Given the three points \( P(1, 0, 1) \), \( Q(2, 0, 0) \), and \( R(-1, 2, 2) \)

a) Find the area of the triangle having \( P \), \( Q \) and \( R \) as vertices.

b) Find the angle at the vertex \( P \) of the triangle having \( P \), \( Q \) and \( R \) as vertices.

c) Find the equation of the plane containing the points \( P \), \( Q \) and \( R \).
(15) 6) A mass distribution occupies the region which is between the spheres
\[ x^2 + y^2 + z^2 = 1 \] and \[ x^2 + y^2 + z^2 = 4. \] If the mass density function is \( \delta(x, y, z) = z^4 \) units of mass/unit volume, find the total mass.
(15) 7) Find the volume of the 3-dimensional region enclosed by the surfaces $y = x^2$, $y = 4$, $z = y$ and $z = 10 - y$.

(15) 8) Calculate the area of that part of the surface $z = y^2$ which is above the region of the $xy$-plane bounded by $y = x$, $x = 0$ and $y = 1$. 
(15) 9) Evaluate the surface integral $\iiint_S x \, dS$ where $S$ is the parametrized surface

$$x = s \cos t, \quad y = s \sin t, \quad z = t, \quad 0 \leq s \leq 4, \quad 0 \leq t \leq \frac{\pi}{4}.$$
The force field $\vec{F} = y^2 \vec{i} + xy \vec{j}$ acts on an object as it moves in the plane. Suppose the object is moved from (0, 0) to (3, 6) by first going along $y = x^2$ from (0, 0) to (2, 4) and then along $y = 2x$ from (2, 4) to (3, 6). Calculate the work done by $\vec{F}$. 
(20) 11) Let $S$ be the closed surface, consisting of two pieces, which bounds the 3-dimensional region which is under $z = 4 - x^2 - y^2$ and above the $xy$-plane. Suppose $\vec{F} = x\vec{i} + y\vec{j} + e^z\vec{k}$. Verify the divergence theorem in this case by calculating both sides of the equation and seeing that they are equal.
(20) 12) Verify Stokes’ theorem by directly calculating both sides of the equation where the vector field is \( \vec{F} = y\vec{i} - x\vec{j} + z\vec{k} \) and the surface is \( x^2 + y^2 + z^2 = 4 \), \( z \geq 0 \).