Calculus III, Summer 2006
Final Exam

You MUST show your work to receive credit.

(20)1. The points P(0,1,1), Q(1,0,1) and R(1,1,0) are the vertices of a triangle in 3-space.

a) Find the angle (in degrees) at the vertex P of the triangle

b) Find the area of the triangle.

c) Find the equation of the plane which contains the triangle.
(20)2. An object is moving in a 3-space according to the parametric equations \( x = 2\cos t, \)
\[ y = 2\sin t, \quad z = t^2, \]
where \( t \) is the time. Find the followings:

a) Position vector \( \mathbf{r} = \)

b) Velocity vector \( \mathbf{v} = \)

c) Acceleration vector \( \mathbf{a} = \)

d) Speed \( = \)

e) \( a_T = \)

f) Curvature \( K = \)

g) \( a_N = \)
(20) 3. An object is moving in the xy-plane along the curve \( y = \frac{1}{2} x^2 \), starting at \( t = 0 \), from left to right. Its speed is \( 2(1 + t^2) \) ft/sec.

a) Find \( a_r \) and \( a_N \) as a function of \( t \) at \( (t, \frac{1}{2} t^2) \).

b) Suppose that at \( t = 1 \) the object is at \((x, y) = (1, 1/2)\). Find the velocity vector and the acceleration vector at this point.
(20)4. Given \( f(x, y) = \frac{1}{y(x^2 + 4y)^2} \).

a) Find the gradient vector field \( \nabla f(x, y) \).

b) Find the directional derivative of \( f(x, y) \) at \((x, y) = (1, 2)\) in the direction of the vector \( \mathbf{v} = 3\mathbf{i} - \mathbf{j} \).

c) Find a vector which points in the direction that gives the largest directional derivative for \( f(x, y) \) at \((x, y) = (1, 2)\).

d) Find the value of the direction of maximum increase of \( f(x, y) \) at \((1, 2)\).
(25)5. Let \( f(x, y) = x^3 y - 12xy + y^2 \). Find all the critical points and then use second partial test to classify these.
(15) 6. Find the volume of the solid bounded by the graphs of the equations
\( y = 3x - x^2, y = 2x, z = 0 \) and \( z = x + 2y \).
(20)7. Use a triple integral in spherical coordinates to calculate the volume of the 3-D region which is bounded by the spheres \( x^2 + y^2 + z^2 = 4 \) and \( x^2 + y^2 + z^2 = 25 \) and above the cone \( z = \left( x^2 + y^2 \right)^{\frac{1}{2}} \).
(20) 8. Let $F(x,y,z) = \langle y^2z^3, 2xyz^3, 3xy^2z^2 \rangle$ be a vector field.

a) Find the divergence of the vector field.

b) Is the vector field $F$ conservative? If so, find the potential function.
(20)9. Use **Green’s Theorem** to find the work done by the force field $F(x,y) = (y^3, x^3 + 3xy^2)$ on a particle that is moving around $C$ where $C$ is a path from $(0,0)$ to $(1,1)$ along the graph of $y = x^3$ and from $(1,1)$ to $(0,0)$ along the graph of $y = x$. 
(20) 10. Verify Stokes’s Theorem for the vector field \( F(x, y, z) = (-y + z, x - z, x - y) \), where \( S \) is the surface of the paraboloid \( z = 4 - x^2 - y^2 \) oriented with the upward pointing normal and \( C \) is the trace of \( S \) in the XY-plane.