(20) 1. The three points $P(1,1,1)$, $Q(2,0,-2)$ and $R(2,2,4)$ determine a triangle in 3-space.

   a) find the angle at the vertex $P$ (in degrees)

   b) calculate the area of the triangle

   c) find the equation of the plane which contains the triangle
(25) 2. An object is moving in 3-space in such a way that its acceleration vector as a function of the time $t$ is

$$a = \cos t \, \mathbf{i} + \sin t \, \mathbf{j} + \mathbf{k}.$$ 

When $t = 0$ its velocity vector is $\mathbf{v}(0) = -\mathbf{j} + \mathbf{k}$.

Find, as functions of $t$,

a) velocity vector

b) the tangential component of acceleration

c) the normal component of acceleration

d) the curvature
(20) 3. Let \( f(x,y) = y \left( x^2 + 2y \right)^{\frac{3}{2}} \).

a) find the gradient vector field of \( f(x,y) \)

b) find the directional derivative of \( f(x,y) \) at the point \( (x,y) = (2,6) \) in the direction which points from \( (2,6) \) toward the point \( (x,y) = (3,4) \)

c) find the value of the largest directional derivative for \( f(x,y) \) at \( (2,6) \)

d) in which direction does the largest directional derivative of part (c) occur?
(15) 4. Suppose that \( z \) is a function of \( x \) and \( y \), \( z = f(x,y) \). Let \((r, \Theta)\) denote polar coordinates in the xy-plane. We can then consider \( z \) to be a function of \( r \) and \( \Theta \).

Suppose that you know that at \((x,y) = (3,4)\) we have 
\[
f_x(3,4) = 5 \quad \text{and} \quad f_y(3,4) = 2.
\]

Use the chain rule to find the values of \( \frac{\partial z}{\partial r} \) and \( \frac{\partial z}{\partial \Theta} \) when \( x = 3 \) and \( y = 4 \).
(15) 5. Use the method of Lagrange multipliers to find the largest value and the smallest value for \( f(x, y) = 3x^4 + y^4 \) on the circle \( x^2 + y^2 = 4 \).
6. Let \( f(x, y) = 3x^2y - 6x^2 + 2y^3 - 36y \).

Find all the critical points of \( f(x, y) \) and then apply the second partials test to determine the nature of each critical point.
(15) 7. A mass distribution occupies the 3D-region in the 1st octant which is enclosed by the coordinate planes and the planes 
    \[ x + z = 1 \quad \text{and} \quad x + y = 1. \]

    The mass density function is \( \delta = 24(x + z) \) units of mass/unit volume. Calculate the total mass.
8. An object moves in 3-space starting at the point \((x, y, z) = (3, 2, 1)\) and stopping at the point \((x, y, z) = (7, 4, 5)\). It moves along the line segment joining these two points.

Throughout its motion it is acted upon by the force field

\[ \mathbf{F} = -y \mathbf{i} + x \mathbf{j} + z \mathbf{k} \, . \]

Calculate the work done by \(\mathbf{F}\).  

9. Let $S$ be the parametrized surface

$$x = s \cos t, \quad y = s \sin t, \quad z = t, \quad 0 \leq s \leq 1 \text{ and } 0 \leq t \leq \pi.$$ 

Evaluate the surface integral

$$\iint_S y \, ds$$
10. Let $Q$ be the 3D-region which is under the plane $z = 4$ and above the paraboloid $z = x^2 + y^2$.

Let $S$ be the boundary of $Q$.

Let $F = x \hat{i} + y \hat{j} + z \hat{k}$ a vector field.

Verify the divergence theorem in this case by calculating both sides of the equation and seeing that they are equal.
(20) 11. Let S be that part of the surface \( z = x^2 + y^2 \) which is under
the plane \( z = 4 \) oriented with the upward pointing normal,
and let C be the boundary of S.

Let \( F = -zy \mathbf{i} + zx \mathbf{j} + z \mathbf{k} \), a vector field.

Verify Stokes's theorem in this case by calculating both sides of
the equation and seeing that they are equal.