25) 1. The points $P(1,1,0)$, $Q(1,0,1)$ and $R(4,1,-4)$ are the vertices of a triangle in 3-space.

   a) Find the angle (in degrees) at the vertex $P$ of the triangle.

   b) Find the area of the triangle.

   c) Find the equation of the plane which contains the triangle.
15) 2. The motion of an object in the plane is given by the position vector function \( \mathbf{r} = 3\cos(t^2) \mathbf{i} + 3\sin(t^2) \mathbf{j} \) for \( t \geq 0 \), where \( t \) is the time.

a) sketch the curve of the motion

b) Find, as functions of \( t \)

velocity vector \( \mathbf{v} = \)

speed =

\( a_T = \)

\( a_N = \)
20) 3. Let \( f(x,y) = x^2 + x^2 y + y^2 \). Suppose that you are standing at the point \((x,y) = (1,1)\).

   a) If you move along a line from \((1,1)\) toward the point \((2,3)\), find the directional derivative of \(f(x,y)\) at \((1,1)\) in your direction of motion.

   b) If you move along a line from \((1,1)\) toward the point \((-1,2)\), find the directional derivative of \(f(x,y)\) at \((1,1)\) in your direction of motion.

   c) Find a vector which points in the direction you should move in order to obtain the largest possible rate of change of \(f(x,y)\) at \((1,1)\).

   d) What is the value of the largest directional derivative of \(f(x,y)\) at \((1,1)\)?
25) 4. Let \( f(x,y) = 3x^2y - 4x^3 + y^3 - 15y \). Find all critical points for \( f(x,y) \) and then apply the second partials test to each critical point to test for local max or min and saddle points.
20) 5. Find the equation of the tangent plane to the given surface at
the specified point.

   a) \[ x^2z + xz^2 + y^2 = yz + 5x^\lambda \] at \((x,y,z) = (1,2,3)\).

   b) the parametric surface \( x = u + v, \ y = u - v, \ z = uv^2 \) at the
point obtained when \( u = 2 \) and \( v = 1 \).
20) 6. Let $S$ be the parametrized surface
\[ x = u, \ y = u^2, \ z = v \] for $0 \leq u \leq 2, \ 0 \leq v \leq 3.$

a) Evaluate the surface integral \[ \iint_S xz \, dS \]

b) Let $F = xi + zj + 4k$. Evaluate the flux integral \[ \iint_S F \cdot N \, dS \]
15) 7. A mass distribution occupies the 3D-region which is between the spheres \( x^2 + y^2 + z^2 = 1 \) and \( x^2 + y^2 + z^2 = 4 \) and is above the xy-plane. If the mass density function is

\[
\delta = (x^2 + y^2)z \quad \text{units of mass/unit volume},
\]

calculate the total mass in the region.
15) 8. The force field \( \mathbf{F} = (x^2 - x^2y) \mathbf{i} + (y^2 + xy^2) \mathbf{j} \) acts on an object as it moves in the plane. Calculate the work done by \( \mathbf{F} \) as the object starts at \((2,0)\) and moves in the counterclockwise direction around \(x^2 + y^2 = 4\) and stops when it gets back to \((2,0)\). Use Green's theorem to evaluate the line integral.
20) 9. Let $Q$ be the 3D-region which is enclosed by the surfaces $y = x^2$, $y = 2x$, $z = y$ and $z = 0$, and $S$ be its boundary. Given the vector field $F = x \mathbf{i} + 2xy \mathbf{j} + z \mathbf{k}$, use the divergence theorem to calculate the outward flux of $F$ across $S$. 
25) 10. Let \( S \) be that part of the plane \( 2x + 2y + z = 2 \) which is in the first octant, oriented with the upward pointing normal. Let \( \mathbf{F} \) be the vector field \( \mathbf{F} = z \mathbf{i} + xy \mathbf{j} + x \mathbf{k} \).

a) \( \text{curl} \ \mathbf{F} = \)

b) Verify Stokes's theorem for this \( S \) and this \( \mathbf{F} \) by calculating both sides of the equation and seeing that they are equal.