(15) 1. Object A starts at the point \((x, y, z) = (1, 2, 3)\) in 3-space and moves along a straight line toward the point \((2, 1, 1)\). Object B starts at \((1, 2, 3)\) and moves along a straight line toward the point \((2, 4, 4)\). Both objects have a nonzero initial speed.

Find the angle (in degrees) between their initial velocity vectors.
(30) 2. An object is moving in 3-space according to the parametric equations
   \[ x = t, \ y = 2t, \ z = 2\sin t, \] where \( t \) is the time.

Find, as functions of \( t \),

a) position vector \( \mathbf{r} = \)

b) velocity vector \( \mathbf{v} = \)

c) acceleration vector \( \mathbf{a} = \)

d) speed =

e) \( a_T = \)

f) curvature \( K = \)

g) \( a_N = \)

When \( t = \frac{\pi}{2} \) find \( T \) and \( N \)
3. Consider the vectors \( \mathbf{a} = i + 3j + 2k \), \( \mathbf{b} = 2i + j \) and 
\( \mathbf{c} = 3i + j + 2k \) placed with their initial points at the origin.

a) Find the area of the parallelogram determined by \( \mathbf{a} \) and \( \mathbf{b} \).

b) Find the volume of the parallelepiped determined by \( \mathbf{a} \), \( \mathbf{b} \) and \( \mathbf{c} \).

c) Find the equation of the plane which contains the arrows representing \( \mathbf{a} \) and \( \mathbf{b} \).
4. Let $f(x,y) = x^3 + x^2y + y^2$.

Suppose that you are moving along a line through the points $(x,y) = (1,2)$ and $(2,1)$ in the direction from $(1,2)$ toward $(2,1)$.

a) Find the directional derivative of $f(x,y)$ at $(1,2)$ in your direction of motion.

b) What is the value of the largest directional derivative of $f(x,y)$ at $(1,2)$?

c) In which direction does the largest directional derivative of part b) occur?
(20) 5. Let $f(x,y) = 6x^2y + 4x^3 - 9y^2 + 12y$.

Find all the critical points of $f(x,y)$ and then use the 2nd partials test to determine the nature of each critical point.
(15) 6. Use the method of Lagrange multipliers to find the largest value and the smallest value of \( f(x, y, z) = 4z + xy \) on the ellipsoid 
\[ x^2 + y^2 + 2z^2 = 18. \]
7. An object moves in 3-space according to the parametric equations
   \[ x = t, \quad y = t^2, \quad z = \sin t \text{ for } 0 \leq t \leq \pi/2. \]

   During its motion it is acted upon by the force field \( F = z \mathbf{i} + x \mathbf{j} + \mathbf{k} \).
   Calculate the work done by the force field \( F \).
8. A mass distribution occupies the region in 3-space which is above 
$z = x^2 + y^2$ and under $z = 4$. The mass density function 
is $\rho = (x^2 + y^2)z$ units of mass/unit volume.

Calculate the total mass.
(25) 9. Let \( S \) be the parametrized surface

\[
x = s \cos t, \quad y = s \sin t, \quad z = s, \quad 0 \leq s \leq 2 \text{ and } 0 \leq t \leq 2\pi.
\]

Find

a) position vector \( \mathbf{r}(s,t) = \)

b) \( \mathbf{r}_s \times \mathbf{r}_t = \)

c) the tangent plane to \( S \) at the point obtained when \( s = 2^{\frac{1}{2}} \)
   and \( t = \frac{\pi}{4} \)

d) the area of \( S \)
10. Let \( Q \) be the solid hemisphere \( x^2 + y^2 + z^2 \leq 4, \quad z \geq 0 \) and \( S \) be its boundary.

If \( F = x^3 z \mathbf{i} + y^3 z \mathbf{j} + z^3 \mathbf{k} \), use the divergence theorem to calculate the outward flux of \( F \) across \( S \).
(25) 11. Let $S$ be that part of the plane $x + y + z = 1$ which is in the first octant oriented with the upward pointing normal vector field. Let $C$ be the positively oriented boundary of $S$.
Suppose that $F = z \mathbf{i} + xy \mathbf{j} + yz \mathbf{k}$. Verify Stokes's theorem in this case by calculating both sides of the equation and seeing that they are equal.