CALCULUS II - EXAM 3
April 10, 2007

Show all work for full credit. No books, notes or calculators are permitted. The point value of each problem is given in the left-hand margin. You have 65 minutes.

(8) 1 Find the equation of the tangent line to the parametric curve $x = \sin(t)$, $y = t^2 + 3t + 2$ at $t = 0$.

(8) 2. Find the arc length of the cycloid $x = t - \sin t$, $y = 1 - \cos t$, $0 \leq t \leq 2\pi$. (Hint: $1 - \cos(t) = 2\sin^2(t/2)$)
(5) 3. a) Sketch the polar curve \( r = 2 \sin \theta \), \( 0 \leq \theta \leq \pi \)

(5) b) Find the slope of the curve in part (a), at an arbitrary angle \( \theta \).

(5) c) Convert the equation of the polar curve in part (a) to a Cartesian equation in \( x \) and \( y \), and identify the curve.
4. Set up the integral for the area of the region enclosed by one loop of the curve $r = \sin(3\theta)$.
  Do not Evaluate the integral.

5. Find the limit of the sequence or explain why it diverges.

   a) $\lim_{n \to \infty} \frac{\ln n}{\sqrt{n}} = $

   b) $\lim_{n \to \infty} \frac{(-1)^n n}{3n - 1} = $
6. a) Let \( S = \sum_{n=1}^{\infty} \frac{1}{n^2} \). Explain why the series converges.

b) The error in approximating \( S \) by the partial sum \( S_n \) satisfies \( |S - S_n| < \int_{n}^{\infty} \frac{1}{x^2} \, dx \). How many terms are needed so that the error is less than .01?

7. Evaluate the series.
\[
\sum_{n=2}^{\infty} \frac{2}{n^2 - n} = 
\]

8. Evaluate the series. Start by writing out the first six terms, so that you can recognize the sum as a familiar series.
\[
\sum_{k=0}^{\infty} \frac{\sin(\pi k/2)}{2^k}
\]
9. Determine whether the following series converge or diverge. State clearly which test you are using and implement the test as clearly as you can. The answer for each problem is worth 2 points and the work you show 5 points.

(7) a) \[ \sum_{n=3}^{\infty} \frac{1}{\sqrt{n-1}} \]

(7) b) \[ \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2} \]
(Problem 9. continued)

(7) c) \[ \sum_{n=1}^{\infty} \frac{n^2 - 1}{5n^2 + 1} \]

(7) d) \[ \sum_{n=1}^{\infty} \frac{n + 5}{5n^3 - n} \]