CALCULUS II - EXAM 3
November 8, 2005

Show all work for full credit. No books, notes or calculators are permitted. The point value of each problem is given in the left-hand margin. You have 70 minutes.

1. Evaluate the improper integral or show that it diverges. Make careful use of limit notation.

   (10) a) \( \int_{0}^{5} \frac{e^x}{e^x - 1} \, dx \)

   (10) b) \( \int_{0}^{\infty} xe^{-x^2} \, dx \)
2. Let \( \{a_n\}_{n=2}^\infty \) be the sequence with \( a_n = \frac{n^2}{n!} \). Show that the sequence is monotone (for \( n \geq 2 \)). Is it increasing or decreasing?

3. Determine whether the following sequence converges or diverges. If it converges find the limit.

\[
\left\{ \frac{\ln n}{\sqrt{n}} \right\}_{n=1}^\infty
\]

4. a) Express the decimal \( .\overline{34} = .343434\ldots \) as a geometric series.

b) Express the decimal in part (a) as a rational number by evaluating the geometric series.

5. Evaluate the series \( \sum_{n=1}^\infty \frac{1}{n-.5} - \frac{1}{n+1.5} \).
6. Use the integral test to determine the convergence or divergence of \( \sum_{n=2}^{\infty} \frac{1}{n(\ln n)^3} \).

7. The sum \( S = \sum_{n=1}^{\infty} \frac{1}{n^3} \) is approximated using the partial sum \( S_N \) with remainder

\[ R_N = S - S_N = \sum_{n=N+1}^{\infty} \frac{1}{n^3}. \]

a) Draw a graph with inscribed rectangles illustrating the inequality \( R_N < \int_N^{\infty} \frac{dx}{x^3} \).

b) In order to approximate \( S \) to four places of accuracy \( (R_N < \frac{1}{2} \times 10^{-4}) \) how large must \( N \) be?

8. The sum \( S = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n^2} \) is approximated using the first 4 terms \( S_4 = .7986 \ldots \)

a) Estimate the error \( R_4 = S - S_4 \) in the approximation.

b) Mark the positions of the partial sums \( S_1, S_2, S_3 \) and the sum \( S \) on the number line below.
9. Determine whether the following series converge or diverge. State clearly which test you are using and implement the test as clearly as you can. The answer for each problem is worth 2 points and the work you show 5 points.

(7) a) \[ \sum_{n=2}^{\infty} \frac{n^2 - 1}{7n^3 + n} \]

(7) b) \[ \sum_{n=1}^{\infty} \frac{1}{n^{1/n}} \]

10. Determine whether the following series converge or diverge. For those that converge state whether the convergence is \textbf{conditional or absolute}. State clearly which test you are using and implement the test as clearly as you can. The answer for each problem is worth 2 points and the work you show 5 points.

(7) a) \[ \sum_{n=1}^{\infty} (-1)^{n+1} \frac{4}{\sqrt{n}(n + 1)} \]

(7) b) \[ \sum_{k=1}^{\infty} \frac{1}{k} \sin\left(\frac{k\pi}{2}\right) \]