Name: ____________________________

You must show all work to receive full credit. No books, notes or calculators are permitted.

The following formulae may be used: \((a > 0)\)

\[
\int \frac{dx}{\sqrt{a^2 - x^2}} = \arcsin\left(\frac{x}{a}\right) + C \\
\int \frac{dx}{\sqrt{x^2 + a^2}} = \ln(x + \sqrt{x^2 + a^2}) + C \\
\int \frac{dx}{a^2 + x^2} = \frac{1}{a} \arctan\left(\frac{x}{a}\right) + C \\
\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} \ln\left|\frac{a + x}{a - x}\right| + C \\
\int \frac{dx}{\sqrt{x^2 - a^2}} = \frac{1}{a} \arccos\left(\frac{|x|}{a}\right) + C \\
\int \frac{dx}{x\sqrt{a^2 + x^2}} = -\frac{1}{a} \ln\left(\frac{a + \sqrt{a^2 + x^2}}{|x|}\right) + C
\]

1. Find the derivatives of the following functions.

[8 points] a). \(y = \ln\left(\frac{x + 1}{x - 1}\right) + 2 \arctan(x)\)

[8 points] b). \(y = \log_2(\sinh(x))\)

[8 points] c). \(y = e^{x^2} \sin(x)\)
[8 points] 2a). Find the derivative of $y = x^x$ using logarithmic differentiation.

[2 points] b) Find the tangent line to $y = x^x$ at the point $x = 1$

[8 points] 3. Implicitly differentiate the following equation with respect to $x$ and solve for $y'$.

$$e^{xy} + x^2 + y^2 = 1$$

[8 points] 4. Find where the function $f(x) = e^{-x^2}$ is increasing and where it is decreasing.
5. Evaluate the following integrals.

[8 points] a). \( \int \frac{x^3 - 6x - 20}{x + 5} \, dx \)

[8 points] b). \( \int_{\pi/6}^{\pi/2} \frac{1}{x \ln x} \, dx \)

[8 points] c). \( \int \frac{\sqrt{x}}{\sqrt{1 + x^3}} \, dx \)

[8 points] d). \( \int \frac{3}{x \sqrt{1 - 9x^2}} \, dx \)
[8 points] 6. Solve the differential equation: \((1 + x^2)y' - 2xy = 0\).

[10 points] 7. Write and solve the differential equation that models the verbal statement. The rate of change of \(P\) is proportional to \(P\). When \(t = 0\), \(P = 5000\) and when \(t = 1\), \(P = 4750\). What is the value of \(P\) when \(t = 5\)?