(45 pts) 1. Evaluate the following three integrals.

(a) \( \int_0^{\pi/2} x \cos x \, dx \)

(b) \( \int \frac{3 \, dx}{(x + 1)(x - 2)} \)
(c) \[ \int \frac{\sqrt{x^2 - 1}}{x} \, dx \] (Nothing but trig substitution will work.)
(15 pts) 2. Find the volume obtained by revolving the region bounded by $y = x^2$, the $y$-axis, and the line $y = 1$, around the $y$-axis.

(10 pts) 3. Same as problem 2, but revolve around the line $y = 1$. 
(15 pts) 4. Consider the curve given parametrically by the equations \( x = t^2 \), \( y = \sqrt{t} \). Make a rough sketch of this curve, for \( 0 \leq t \leq 1 \). Then, set up an integral for the arc-length of this curve \( (0 \leq t \leq 1) \). Do not attempt to evaluate the integral.
(15 pts) 5. Find the centroid of the quarter-disk given by the interior of the circle $x^2 + y^2 = 4$ in the 1st quadrant. (Use the symmetry of this figure, in order to simplify the problem.)
(10 pts) 6. Set up an integral for finding the area of the surface of revolution generated by revolving the curve $y = e^x$ around the $y$-axis $(0 \leq x \leq 1)$.

(10 pts) 7. Change the polar equation $r = 4 \sin \theta$ into an equation in $x$ and $y$, and describe its graph.
8. For each of the infinite series given below, decide if the series converges or if it diverges. Justify your answer.

(a) \[ \sum_{n=2}^{\infty} \frac{1}{n^3 - 1} \]

(b) \[ \int_{n=1}^{\infty} (-1)^{n-1} \frac{1}{\sqrt{n}} \]

(c) \[ \sum_{n=0}^{\infty} \frac{3^n + 2^n}{5^n} \]
(16 pts) 9(a) Find the radius of convergence of the power series
\[ \sum_{n=0}^{\infty} (-1)^n \frac{1}{n} (x - 2)^n. \]

(b) Find the interval of convergence for the power series in part (a).
(15 pts) 10. Compute the Taylor series for the function \( y = \sin x \) at \( a = \pi \).
11. Use your knowledge of MacLaurin series in order to quickly obtain MacLaurin series for the following functions.

(a) $\sin 2x$

(b) $\cos \sqrt{x}$

(c) $\frac{1}{1 + x^3}$

(d) $e^{-x^2}$
(12 pts) 12. Use the result of the problem 12(b) in order to find

$$\int \cos \sqrt{x} \, dx$$

expressed as an infinite series.
Integral Formulas

1. \( \int u^n \, du = \frac{1}{n+1} u^{n+1} + c \) if \( n \neq -1 \).

2. \( \int \frac{du}{u} = \ln |u| + c \)

3. \( \int e^u \, du = e^u + c \)

4. \( \int \sin u \, du = -\cos u + c \quad \int \cos u \, du = \sin u + c \)

5. \( \int \sin^2 u \, du = \frac{1}{2} [u - \sin u \cos u] + c \quad \int \cos^2 u \, du = \frac{1}{2} [u + \sin u \cos u] + c \)

6. \( \int \sec u \, du = \ln |\sec u + \tan u| + c \)

7. \( \int \csc u \, du = \ln |\csc u - \cot u| + c \)

8. \( \int \tan u \, du = \ln |\sec u| + c \quad \int \cot u \, du = \ln |\sin u| + c \)

9. \( \int \sin^n u \, du = -\frac{1}{n} \sin^{n-1} u \cos u + \frac{n-1}{n} \int \sin^{n-2} u \, du + c \)

10. \( \int \cos^n u \, du = \frac{1}{n} \cos^{n-1} u \sin u + \frac{n-1}{n} \int \cos^{n-2} u \, du + c \)

11. \( \int \sec^n u \, du = \frac{1}{n-1} \sec^{n-2} u \tan u + \frac{n-2}{n-1} \int \sec^{n-2} u \, du + c \), if \( n > 1 \).

12. \( \int \tan^n u \, du = \frac{1}{n-1} \tan^{n-1} u - \int \tan^{n-2} u \, du + c \), if \( n > 1 \).

13. \( \int \csc^n u \, du = -\frac{1}{n-1} \csc^{n-2} u \cot u + \frac{n-2}{n-1} \int \csc^{n-2} u \, du + c \), if \( n > 1 \).

14. \( \int \cot^n u \, du = -\frac{1}{n-1} \cot^{n-1} u - \int \cot^{n-2} u \, du + c \), if \( n > 1 \).

15. \( \int \sinh u \, du = \cosh u + c \quad \int \cosh u \, du = \sinh u + c \).

Trigonometry Formulas

1. \( \sin^2 u = \frac{1}{2} (1 - \cos 2u) \)

2. \( \cos^2 u = \frac{1}{2} (1 + \cos 2u) \)

3. \( \tan^2 u + 1 = \sec^2 u \)