Mr. This B. Test III.

1. Evaluate each of the following integrals:

   (8 pts) a. $\int \frac{5}{\sqrt{3x}} + \cos x \, dx$

   (8 pts) b. $\int \frac{\cos 5u}{\sin^3 5u} \, du$

   (8 pts) c. $\int x(x^3 + 1)^2 \, dx$

   (8 pts) d. $\int \frac{\sec^2(\sqrt{x})}{\sqrt{x^2}} \, dx$

   (8 pts) e. $\int t^3 \sqrt{1 + t^2} \, dt$
2. Let \( f(x) = \frac{2x^3 - 4x + 6}{x^2 - 1} \).

(3 pts) a. What are the equations of the vertical asymptotes of \( f(x) \)?

(3 pts) b. What is the equation of the horizontal (slant) asymptote of \( f(x) \)?

(3 pts) c. What is the \( x \)-value of the point(s) (if any) where \( f(x) \) crosses an asymptote?

(3 pts) d. Sketch a graph of \( f(x) \) including all asymptotes.
(15 pts) 3. Find the area bounded by $y = 4x - x^2$ and $x + y = 0$. (You may leave integrals in your answer, e.g., $A = \int_{-\frac{7}{2}}^{2}(x^4 + 5)dx + \int_{1}^{3} x^3 \, dx$.)
4. The curves

\[ xy = k \]  \hspace{1cm} (1)

are graphed for various values of \( k \) below.

In this problem, you will find the equation of the curve which is perpendicular to \( xy = k \) which also passes through the point \((5, 3)\).

Using implicit differentiation we obtain

\[ y + x \frac{dy}{dx} = 0 \quad \text{or} \quad \frac{dy}{dx} = -\frac{y}{x} \]

from equation (1).

The slope of a curve perpendicular to (1) is then given by:

\[ \frac{dy}{dx} = \frac{x}{y}. \]

Solve this differential equation using the point \((5, 3)\) to obtain the equation of the perpendicular curve.
5. Let \( f(x) = 2x^{9/7} - 9x^{2/7} \).

Note that \( f'(x) = \frac{18}{7}x^{2/7} - \frac{18}{7}x^{-5/7} \) and \( f''(x) = \frac{36}{49}x^{-5/7} + \frac{90}{49}x^{-12/7} \).

(3 pts) a. Find every point where \( f'(x) \) is undefined. Label your answers with the number of the figure which best describes the local behavior of the graph near the point where \( f'(x) \) is undefined.

i) ii) iii) iv)

(3 pts) b. Is \( f(x) \) increasing or decreasing near \( x = 10^{-7} \) ?

(3 pts) c. Is \( f(x) \) concave up or concave down near \( x = 10^{-7} \) ?
(4 pts) 6.a) Let \( F(x) = \left( \int_0^x \cos \sqrt{t} \, dt \right) \cos(\sqrt{x}) \). Find \( F'(x) \).

b. Given that \( \int_{-2}^2 f(x) \, du = 1 \), \( \int_0^4 f(x) \, dx = 2 \) and \( \int_0^2 f(x) \, dz = -1 \), compute

(2pts) i) \( \int_{0}^2 f(z) \, dz \),

(2pts) ii) \( \int_{2}^4 f(p) \, dp \),

(2pts) and iii) \( \int_{-2}^0 f(w) \, dw \)

(3 pts) c. Compute \( \int_{-1}^2 x^2(x^2 + 1) \, dx \). (You do not have to simplify.)
(extra 10 pts) 7. A box with a square base is inscribed in a cone of height 6 cm and radius 2 cm. What is the length of the base of the box with the largest volume?