CALCULUS I - EXAM III
November 16, 2004

Show all work for full credit.

(8) 1. Find \( dy \) for the given function \( y = f(x) \). Do not simplify.
   a) \( y = \sqrt{1 + \sin \sqrt{x}} \).
   b) \( y = (1 + (2 + 3x)^{2^2})^{\frac{1}{2}} \).

(6) 2. Set up Newton’s iteration scheme to approximate the \( x \) coordinate of the intersection of the graphs of \( f(x) = x^4 \) and \( g(x) = x + 3 \). Do not simplify.

(6) 3. The diameter \( d \) of a sphere is measured to be 18 cm with a maximum possible error of 0.05 cm. Use differentials to approximate the possible propagated error in calculating the surface area of the sphere. (The area of a sphere of radius \( r \) is \( A(r) = 4\pi r^2 \).)
4. Evaluate the indefinite integrals.
   a) \[ \int \left( \frac{1}{x^2} - \sqrt{x} + 2 \right) \, dx = \]
   
   b) \[ \int 3 \sin(5x) \, dx = \]
   
   c) \[ \int \left( \sec^2(t) - \sin(3t) \right) \, dt = \]

5. Find the function \( y = f(x) \) satisfying the initial value problem:
   \[ f''(x) = x^{-2/3}, \quad f'(4) = 2, \quad f(0) = 6. \]

6. a) Write the following sum in expanded notation but do not evaluate it.
   \[ \sum_{j=1}^{4} (-1)^j \frac{j}{2j + 1} = \]
   
   b) Write the following sum in summation (sigma) notation.
   \[ 3 + 7 + 11 + 15 + 19 + \ldots + 99 + 103 = \]
7. a) Write out the Riemann sum \( \sum_{i=1}^{n} f(c^*_i) \Delta x \) for \( f(x) = x^2 \) on the interval \([0,2]\) using a regular partition into \( n = 4 \) subintervals with the chosen point \( c^*_i \) in the \( i^{th} \) interval taken as the right endpoint. (Do not evaluate the sum.)

b) Write the definite integral that the Riemann sum in part (a) approximates, and evaluate it.

8. Evaluate the definite integrals:
   a) \( \int_{-\pi/2}^{\pi/2} (2t + \cos t) \, dt \) =

   b) \( \int_{1}^{4} \frac{u - 2}{\sqrt{u}} \, du \) =

9. Sketch the region bounded by the graphs of the equations \( y = x^3 + x, \ x = 2, \) and \( y = 0, \) and find its area.
10. A ball is thrown vertically upward from a height of 6 ft with an initial velocity of 60 ft per second. How high will the ball go? (The acceleration due to gravity is \(-32 \text{ ft}/\text{sec}^2\).)

10. Let \( g(x) = \int_0^x f(t)dt \) where \( f(x) \) is the function graphed at left below. Sketch the graph of \( g(x) \) over the interval \([0,3]\) at right and find the following.

a) \( f'(2) = \quad \) b) \( g(1) = \quad \) c) \( g(3) = \quad \) d) \( g'(2) = \quad \)