CALCULUS I - EXAM II
October 26, 2004

Show all work for full credit. You may not use a calculator, nor any books or notes. The point value of each problem is given in the left-hand margin.

(12) 1. Calculate the limits

a) \[ \lim_{x \to +\infty} \frac{3x}{\sqrt{x^2 + 2}} = \]

b) \[ \lim_{x \to -\infty} \frac{3x}{\sqrt{x^2 + 2}} = \]

c) Write the equations of the horizontal asymptotes to the graph of \( f(x) = \frac{3x}{\sqrt{x^2 + 2}} \).

(12) 2. Let \( f(x) = \frac{3x^2 + x + 1}{x^2 - 1} \). In a) and b) below be sure to show all your work!

a) Evaluate \( \lim_{x \to +\infty} f(x) = \)

b) Evaluate \( \lim_{x \to -\infty} f(x) = \)

c) Give the equation of the horizontal asymptote for the graph of \( f(x) \).

d) Give the equations of all vertical asymptotes for the graph of \( f(x) \).
3. Calculate \( \lim_{x \to \infty} \frac{5 \cos x}{x} = \) 

4. Locate the absolute extrema of the function \( g(t) = \frac{t}{t - 2} \) on the closed interval \([3, 5]\).

5. a) State the mean value theorem: If \( f(x) \) is continuous on \([a, b]\) and ......................... on \((a, b)\), then there exists a point \( c \) in \((a, b)\) such that ............................................................

b) Show that the function \( f(x) = \sqrt{2 - x} \) satisfies the hypothesis of the mean value theorem on the interval \([-7, 2]\), and find all numbers \( c \) that satisfy the conclusion of that theorem.

6. Let \( f(x) = (1 - x)(\sqrt[3]{x}) \).
   a) Calculate \( f'(x) \) and simplify your answer (in factored form).

   b) List the critical numbers of \( f(x) \):

   c) Draw a number line to indicate the open intervals where \( f(x) \) is increasing and decreasing.

   d) Classify each of the critical numbers of \( f(x) \) as either a local minimum, local maximum or neither.
7. Determine the points of inflection of the function \( h(s) = s + \cos s \) that lie in the interval \([0, 2\pi]\).

8. Sketch the graph of the function \( g(x) = x^3 - 3x^2 + 3 \) as follows:
   a) Give the exact values of all critical points and draw a number line showing where \( g(x) \) is increasing and decreasing.
   b) Evaluate: \( \lim_{x \to +\infty} g(x) = ........................................, \lim_{x \to -\infty} g(x) = .................................
   c) Give the exact \( x \)-coordinates of all inflection points of \( g(x) \). Draw a number line indicating where the graph of \( g(x) \) is concave up and concave down.
   d) In the table below, give both coordinates of all local minima and maxima.
   e) Sketch the graph of \( g(x) \).

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\( y \)-axis

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9. Determine the dimensions of a rectangular solid (with a square base) of maximum volume if its surface area is 150 square inches.