(25 pts.) 1. Find each limit.

(a) \( \lim_{x \to 3^+} \frac{6}{x - 3} \)

(b) \( \lim_{x \to \pi^+} \csc x \)

(c) \( \lim_{s \to -2} \frac{s^3 + 8}{(s + 2) s} \)

(d) \( \lim_{t \to -3} \frac{\sqrt{t + 1} - 2}{t - 3} \)

(e) \( \lim_{t \to 2} t^2 - 2 + \frac{1}{t^2} \).
2. Find the equation of the tangent line to the graph of \( y = x^3 + 2x - 7 \) at the point when \( x = 2 \).

3. Using the definition of the derivative as the limit of a difference quotient, find the derivative of the function \( f(x) = \frac{1}{x + 3} \). You will not receive credit if you solve this using the rules which we later developed.

4. Explain why the function \( y = \sin x + 2 \cos x \) has a zero in the interval \( \left[ \frac{\pi}{2}, \pi \right] \).
5. Let \( f(x) = \frac{x^2 + x - 6}{2x^2 + x} \).

(a) What is the domain of \( f(x) \)?

(b) Find the horizontal asymptotes of the graph of \( f \).

(c) Find the vertical asymptotes of the graph of \( f \).

(d) At which points is \( f \) continuous?

6. Consider the function \( f(x) \) given by:

\[
 f(x) = \begin{cases} 
  x & \text{if } x < 0 \\
  e^x & \text{if } x \geq 0 
\end{cases}
\]

Explain why the function is discontinuous at \( x = 0 \). Sketch a graph of the function.
(5pts.) 7. Show that the equation $x^3 = -x - 1$ has a solution between $-1$ and $0$.

(10 pts.) 8. Find the derivative of each function. You may use the rules we later developed for differentiation.
(a) $y = \frac{1}{x^7} + \pi^7$

(b) $f(t) = \frac{t^2}{t + e^t}$

(10 pts.) 9. Sand is pouring into a conical pile in such a way that the height of the sand pile is always 1.5 times the radius. The radius is increasing at the rate of 2 meters per second. How quickly is the volume changing at the instant when the radius is 10 meters?
Show all your work in the space under each question. Please write legibly and organize your solutions in a logical and coherent form; answers which are illegible or confusing will not receive credit. You may not use notes, books or calculators. The point value of each problem is listed next to the problem number.

(10 pts.) 1. Let \( f(t) = t^2 e^{2t} \).
   (a) Find \( f'(t) \).

   (b) Find \( f''(t) \).

(7 pts.) 2. Find the equation of the tangent line to the graph of \( x^3 + y^3 = 35 \) at the point (2,3).

(8 pts.) 3. Find the absolute maximum and absolute minimum of the function \( f(x) = \frac{x}{x^2 + 1} \) on the interval \([0, 2] \).
4. A trough is 10 m long. Its ends have the shape of an equilateral triangle with sidethths 2 m and so that a vertex of the triangle is pointed down. If water is entering the trough at the rate of 12 m³ per minute, how quickly is the water level rising when the water is 1 m deep?

5. Explain, quoting any relevant theorems, why the inequality

\[ | \cos x - \cos y | \leq | x - y | \]

is valid for all real numbers \( x \) and \( y \).

6. Find \( \lim_{x \to -\infty} xe^{-2x} \).

7. Use differentials (or equivalently, a linear approximation) and your knowledge of \( \tan 45^\circ \) to estimate the given number: \( \tan(46^\circ) \).
8. Find the derivative of each function.

(a) \( f(x) = \ln(\tan x) \)

(b) \( y = \log_{10}(\sec x + \tan x) \).

(c) \( f(x) = x^x \).

(d) \( f(x) = e^x(\sin x + x) \).

(e) \( y = \tan^{-1}(e^x) \).
(5 pts.) 9. Find the limit \( \lim_{\theta \to 0} \frac{\tan(3\theta)}{4\theta} \).

(20 pts.) 10. Consider the function \( f(x) = (x^2 - 4)^2 \).
(a) Find the intervals of increase or decrease.

(b) Find the local maximum or minimum values.

(c) Find the intervals of concavity.

(d) Find the inflection points.

(e) Use the information from parts (a)-(d) to sketch a graph of the function.
(25 pts.) 1. Consider the function \( f(x) = \frac{x}{x - 4} \).

(a) Find the critical numbers of \( f \).

(b) Find the intervals on which \( f \) is increasing and decreasing.

(c) Find the intervals on which \( f \) is concave up and concave down and the points of inflection.

(d) Find all asymptotes of the graph of \( f \).

(e) Graph \( f \). Incorporate in your graph all the information you found in parts (a)-(d) as well as the intercepts of the function. Label intercepts, asymptotes, local maximums, local minimums, and points of inflection on your graph.
(10 pts.) 2. A package sent by the United States Postal Service can have a maximum combined length and girth (perimeter of a cross section) of 108 inches. Assuming the cross section of the package is square, what are the dimensions of the package of maximum volume which can be sent?

(8 pts.) 3. A soccer team plays in a stadium that holds 50,000 spectators. With ticket prices at €45 attendance was 30,000. With ticket prices at €35, attendance was 40,000. Assuming it is linear, find the demand function. Determine what ticket price would maximize revenue.

(7 pts.) 4. Use Newton's method with initial approximation $x_1 = 2$ to find $x_3$, the third approximation to the root of the equation $x^2 - 2 = 0$. 
(8 pts.) 5. Find the most general antiderivative of each function.

(a) \( f(x) = 4 + x^2 - x^4 \)

(b) \( g(x) = \sec^2 x + \cos x \)

(8 pts.) 6. Compute the integrals.

(a) \( \int_1^8 \frac{1}{\sqrt{x}} \, dx \)

(b) \( \int_0^4 e^y + y^3 \, dy \)

(9 pts.) 7. A ball is dropped from the top of a building and hits the ground at the speed of 256 \( \frac{ft}{sec} \). How tall is the building? Assume that the acceleration due to gravity is \( -32 \frac{ft}{sec^2} \) and that there is no air resistance.
(8 pts.) 8. Use the midpoint rule with \( n = 4 \) to approximate the integral \( \int_0^2 x^2 \, dx \).

(4 pts.) 9. Find the derivative of the function \( F(x) = \int_{\pi/4}^{x} \tan 2t \, dt \).

(4 pts.) 10. Suppose \( f \) is a function which has \( \int_{2}^{5} f(x) \, dx = 3 \) and \( \int_{4}^{5} f(x) \, dx = \frac{3}{2} \). Find \( \int_{2}^{4} f(x) \, dx \).

(9 pts.) 11. Find two positive numbers whose product is 100 and whose sum is a minimum.
Show all your work in the space under each question. Please write legibly and organize your solutions in a logical and coherent form; answers which are illegible or confusing will not receive credit. You may not use notes, books or calculators. The point value of each problem is listed next to the problem number.

(25 pts.) 1. Find each limit.

(a) \( \lim_{x \to 3^+} \frac{6}{x - 3} \)

(b) \( \lim_{x \to \pi^+} \csc x \)

(c) \( \lim_{t \to 3} \frac{\sqrt{t+1} - 2}{t - 3} \)

(e) \( \lim_{\theta \to 0} \frac{\tan(3\theta)}{4\theta} \).

(f) Find \( \lim_{x \to \infty} \frac{x^2}{e^{2x}} \).
(10 pts.) 2. Find the equation of the tangent line to the graph of \( y = x^3 + 2x - 7 \) at the point when \( x = 2 \).

(10 pts.) 3. Using the definition of the derivative as the limit of a difference quotient, find the derivative of the function \( f(x) = \frac{1}{x + 3} \). You will not receive credit if you solve this using the rules which we later developed.

(5 pts.) 4. Use differentials (or equivalently, a linear approximation) to estimate the given number: \( \sin(31^\circ) \).
(5 pts.) 5. Show that the equation $x^3 = -x - 1$ has a solution between $-1$ and 0.

(7 pts.) 6. Find the equation of the tangent line to the graph of $xy + y^3 = 10$ at the point $(1, 2)$.

(8 pts.) 7. Find the absolute maximum and absolute minimum of the function $f(x) = 3x^4 - 4x^3$ on the interval $[-1, 2]$.

(5 pts.) 8. Suppose $f$ is a function which has $\int_2^5 f(x)dx = 3$ and $\int_4^5 f(x)dx = \frac{3}{2}$.

Find $\int_2^4 f(x)dx$. 
(9 pts.) 9. The *Pequod* is approaching a tiny island from the south at $15\text{ km/hr}$. The *Hispaniola* is sailing away from the island, toward the west at $10\text{ km/hr}$. How quickly is the distance between the ships changing if the *Pequod* is 5 km from the island, and the *Hispaniola* is 12 km from the island?

(8 pts.) 10. Find the volume of the solid generated when the region bounded by $y = \sqrt{x}$, $x = 2$, $x = 4$, and $y = 0$ is revolved around the $x$-axis.

(8 pts.) 11. Find the volume of the solid generated when the region bounded by the curves $y = 1 + x^2$, $y = 0$, $x = 1$, and $x = 2$ is revolved around the $y$-axis.
Find the derivative of each function.

(a) \( f(x) = e^{\tan x} \)

(b) \( y = \ln(\sec x + \tan x) \).

(c) \( f(x) = x^{\ln x} \).

(e) \( y = \tan^{-1}(x^2) \).

(f) \( F(x) = \int_{\frac{x}{4}}^{x} \tan 2t \, dt \).
(25 pts.) 13. Consider the function \( f(x) = \frac{x}{(x + 1)^2} \)

(a) Find the critical numbers of \( f \).

(b) Find the intervals on which \( f \) is increasing and decreasing.

(c) Find the intervals on which \( f \) is concave up and concave down and the points of inflection.

(d) Find all asymptotes of the graph of \( f \).

(e) Graph \( f \). Incorporate in your graph all the information you found in parts (a)-(d) as well as the intercepts of the function. Label intercepts, asymptotes, local maximums, local minimums, and points of inflection on your graph.
14. A box with an open top is to be made from a square sheet of cardboard 1 m square by cutting out squares from each of the four corners and bending up the sides. Find the largest volume that such a box can have.

15. Find the area of the region bounded by the curves $y = x^2$, $y = x^{\frac{1}{2}}$, $x = 0$ and $x = 1$.

16. Estimate the area under the graph of $f(x) = \sin x$ between 0 and $\pi$ using four approximating rectangles and left hand endpoints.
(8 pts.) 17. (a) Find the most general antiderivative of \( g(x) = \sec x \tan x \).

(b) Compute \( \int_0^4 e^y + y^2 \, dy \).

(8 pts.) 18. Use Newton’s method with initial guess \( x_1 = 1 \) to find the approximation \( x_3 \) of \( \sqrt[3]{2} \).

(9 pts.) 19. A ball is dropped from the top of a building and hits the ground 4 sec later. How tall is the building? Assume that the acceleration due to gravity is \( -32 \frac{ft}{sec^2} \) and that there is no air resistance.