1 (total points: 12). Differentiate:

(a)\(6\) points \(f(x) = 5^x + x^{1/3} - 2\ln x\).

(b)\(6\) points \(g(t) = Ct^2 - 7t^5 - 1/\sqrt{t}\).

2 (total points: 12). (a)\(4\) points Find the derivative of \(f(x) = (1 + x^2)^{2006}\).

(b)\(4\) points Find the derivative of \(f(x) = e^{x^3-1}\).

(c)\(4\) points Find the derivative of \(f(x) = 8\sqrt{\ln(x)}\).
3 (total points: 12). (a)(6 points) Differentiate \( P(t) = e^{2t} \cdot \ln(t) \) and simplify.

(b)(6 points) Find the derivative of \( y = \frac{x + 10}{1 - x^2} \) and simplify it.

4 (total points: 12). Find the value of \( a \) so that the function \( f(x) = x \cdot e^{ax} \) has a critical point at \( x = 3 \).
5 (total points: 13). Find the global maxima and global minima of the function $f(x) = x^5 - 2 \cdot x^3$ on the interval $[-1, 2]$.

6 (total points: 13). The total cost in dollars of producing $q$ units of a laptop is given by $C(q) = q^3 - 60q^2 + 1400q + 1000$ for $0 \leq q \leq 50$.

(a) (2 points) What are the fixed costs?

(a) (4 points) Determine the profit function $\pi(q)$ if the laptop sells for $788$ per unit.

(a) (7 points) Find the production level for which the profit is maximized and give the corresponding profit.
7 (total points: 13). The average cost in euros for producing a certain product is given by \( a(q) = q^2 - 6q + 15 \), where \( q \) is in thousands and \( 0 \leq q \leq 5 \).

(a) (4 points) Determine the total cost function \( C(q) \).

(b) (9 points) Find the minimum average cost and the corresponding value of \( q \).

8 (total points: 13). The bird flu has reached Germany after 100 infected swans arrived at the island of Ruegen. It is expected that the number of birds infected can be modeled by a logistic growth function

\[
P(t) = \frac{2,500,000}{1 + C e^{-3t}},
\]

where \( t \) is the number of weeks since the arrival of the swans.

(a) (3 points) How many birds will become infected in the long run under this model?

(b) (5 points) Determine the remaining parameter \( C \) of \( P(t) \).

(c) (5 points) When is the rate of newly infected birds expected to be the highest?