Math 205  Final Exam  
7:00p.m.–8:50p.m., Wednesday, May 10, 2006

Instructor:_________________  Class Time:__________  Name:_________________

No books or notes are allowed. Please read the problems carefully and do all you are asked to do. You must show your work! You can use the back page as a scratch paper. Do the problems at the space provided.

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1 (total points: 12). A $20,000 car depreciates linearly down to $4,000 in 8 years.

(a)(6 points) Find the formula for the value $y$ of the car after $x$ years for $0 \leq x \leq 8$.

(b)(6 points) After how many years is the value of the car the half of its original price?

2 (total points: 15). Find the derivative of the following functions:

(a)(5 points) $y = \ln(1 + 3x^2)$.

(b)(5 points) $t = s^3 \cdot e^{s-1}$.

(c)(5 points) $w = \frac{5u^2}{1-2u}$. 
3 (total points: 15). The demand and supply curves for a certain product are given in terms of price, \( p \) (in dollars), by

\[
D(p) = 2500 - 20p \quad \text{and} \quad S(p) = 10p - 500.
\]

(a)(6 points) Find the equilibrium price and quantity.

(b)(6 points) If a specific tax of $6 per unit is imposed on suppliers, find the new equilibrium price and quantity.

(c)(3 points) How much of the $6 tax is paid by the suppliers and by the consumers, respectively?

4 (total points: 14). A person likes to put $3,000 on a saving account. Bank A is paying a 3\% nominal annual rate compounded every three months. Bank B is offering a 2.9\% nominal annual rate compounded continuously.

(a)(7 points) Determine the amount in the account after 5 years for bank A.

(b)(7 points) How long does it take the balance in the account of bank B to double?
5 (total points: 12). The following information about a function \( f(x, y) \) is given: \( f(-10, 3) = 7.5 \), \( f_x(-10, 3) = 0.3 \), and \( f_y(-10, 3) = -1.2 \). Estimate \( f(-10.5, 3.4) \).

6 (total points: 15).
(a)(5 points) Find the partial derivative \( \frac{\partial f}{\partial y} \) of \( f(x, y) = 3x^4y^2 - x^2y + 7 \).

(b)(5 points) Find the partial derivative \( g_x \) of \( g(x, y) = \frac{\sqrt{x}}{y^2} \).

(c)(5 points) Find the partial derivative \( h_{st} \) of \( h(s, t) = \ln(s + t) \).
7 (total points: 16). The total cost in dollars, $C(q)$, of producing $q$ commercial dish washers is given by $C(q) = q^3 - 66q^2 + 1,500q + 10,000$.

(a) (3 points) What is the fixed cost?

(b) (3 points) Determine the profit function, $\pi(q)$, if each washer sells for $1500.

(c) (10 points) What is the maximum total profit?

8 (total points: 14). (a) (7 points) Find all the critical points $(a, b)$ of the function

$$f(x, y) = 3x^2 + y^2 + 4xy - 6x - 4371.$$ 

(b) (7 points) Use the $D$-test to decide for each of the critical points $(a, b)$ found in part (a) if it is a local minimum, a local maximum, or neither. $(D = f_{xx}(a, b)f_{yy}(a, b) - f_{xy}(a, b).)$
9 (total points: 15). A company manufactures $x$ units of item $A_1$ and $y$ units of item $A_2$. The total cost in dollars, $C$, of producing these two items is approximated by the function $C = 5x^2 + 2xy + 3y^2 + 800$.

(a)(10 points) If the production quota for the total number of items (both types combined) is 39, find the minimum production cost by using the Lagrange multiplier method.

(b)(5 points) Estimate the additional production cost if the production is raised to 41.

10 (total points: 15). Find the inverse of the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$

using basic row-operations.
11 (total points: 12). Find all values for $x, y$ and $z$ such that the following matrix equation is satisfied:

\[
\begin{bmatrix}
10 & x + y + 8 \\
-z & x - y
\end{bmatrix} = 
\begin{bmatrix}
10 & 2x + 3y \\
z^2 & y
\end{bmatrix}.
\]

12 (total points: 15). Given are the matrices

\[A = \begin{bmatrix}
1 & 2 & -3 \\
4 & -5 & 6
\end{bmatrix}, \quad B = \begin{bmatrix}
4 & 0 & -1 \\
7 & 1 & 0
\end{bmatrix}\quad \text{and}\quad C = \begin{bmatrix}
-3 & 3 \\
1 & 3 \\
1 & 0
\end{bmatrix}.
\]

Compute the following matrix expressions or indicate that this is impossible:

(a) (5 points) $8A - 4(B + A) =$

(b) (5 points) $A(-C) =$

(c) (5 points) $C(-B) =$
13 (total points: 12). Solve the following system of equations by the inverse matrix method (no credit for other solutions!):

\[
\begin{align*}
    x + y - z &= 18 \\
    3x - y &= -27 \\
    2x - 3y + 4z &= 9
\end{align*}
\]

The inverse matrix of \( A = \begin{bmatrix} 1 & 1 & -1 \\ 3 & -1 & 0 \\ 2 & -3 & 4 \end{bmatrix} \) is \( A^{-1} = \begin{bmatrix} 4/9 & 1/9 & 1/9 \\ 4/3 & -2/3 & 1/3 \\ 7/9 & -5/9 & 4/9 \end{bmatrix} \).

14 (total points: 18). The following table describes the interrelationship between the production of two industries P and Q as well as the external demand of their products in a given year.

<table>
<thead>
<tr>
<th>Demand of P</th>
<th>Demand of Q</th>
<th>External Demand</th>
<th>Total Output</th>
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</thead>
<tbody>
<tr>
<td>Production of P</td>
<td>6</td>
<td>30</td>
<td>24</td>
</tr>
<tr>
<td>Production of Q</td>
<td>30</td>
<td>40</td>
<td>50</td>
</tr>
</tbody>
</table>

If the forecast for the external consumer demand in the next year is \( D' = \begin{bmatrix} 57 \\ 57 \end{bmatrix} \) what should the future total output \( X' = \begin{bmatrix} x' \\ y' \end{bmatrix} \) be?