1 (12 pts). The quantities of demand and supply functions for a certain product are given in terms of price, \( p \), by \( D(p) = 2500 - 20p \) and \( S(p) = 10p - 500 \). Find the equilibrium price \( p \) and quantity. (You need not draw a graph.)

2 (13 pts). Find the derivative of the function \( y = 4x^3 + 3x^2 - 5x \) at \( x = 2 \).
3 (15 pts). Find derivatives of following functions.
   a. \( y = z^2 + \frac{1}{z} \)

   b. \( s = 5t + \ln(t + 2) \)

   c. \( w = \frac{3z - 1}{5 + z} \).

4 (10 pts). Find the equation of the tangent line to the function of \( f(x) = x^3 + 1 \) at \( x = 1 \). (Do not draw the graph)
5 (15 pts). The total cost of the production of $q$ units of certain product is $C(q) = 10,000 + 3q^2$, and the product sells for 450 per unit.

- (1). Find the revenue function $R(q)$ and profit function $P(q)$.

- (2). At what quantity $q$ is profit function $P(q)$ maximized? (Show your work in details.)

6 (10 pts). Find partial derivatives of following functions:

- (a. 5pts). \( \frac{\partial Q}{\partial x} \) if $Q(x, y) = 5x^2y - 3xy^3$

- (b. 5pts). $f_x$ if $f(x, y) = 5e^{xy}$. 

7 (12 pts). The following table gives some data of a function $f(x, y)$:

<table>
<thead>
<tr>
<th>$y$</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>98</td>
<td>91</td>
<td>85</td>
<td>81</td>
</tr>
<tr>
<td>6</td>
<td>103</td>
<td>98</td>
<td>92</td>
<td>88</td>
</tr>
<tr>
<td>8</td>
<td>112</td>
<td>105</td>
<td>99</td>
<td>93</td>
</tr>
</tbody>
</table>

(a.) Estimate $f_x(10, 6)$ and $f_y(10, 6)$

(b.) Estimate the value of $f(15, 7)$ using the local linearity: $\Delta f \approx f_x \Delta x + f_y \Delta y$.
(Hint: set $\Delta f = f(15, 7) - f(10, 6)$).

8 (13 pts). Let $f(x) = 2x - \ln x$.

(a. 6 pts). Find the critical point(s) of $f(x)$.

(b. 7 pts). Find the global maximum and minimum of $f(x)$ over the interval $1 \leq x \leq 3$.
(Show your work.)
9 (13 pts). Let \( f(x, y) = x^2 + y^2 - xy - 3y + 30 \).

(a. 7pts). Find the critical point \((a, b)\) of the function \( f(x, y) \).

(b. 6pts). Apply the test function \( D = f_{xx}(a, b)f_{yy}(a, b) - f_{xy}^2(a, b) \) and the sign of \( f_{xx} \) to determine whether the critical point is a local minimum, local maximum, or neither.

10 (12 pts). Apply the method of Lagrange multipliers to solve following problem. A company manufactures \( x \) units of one item and \( y \) units of another. The total cost (in dollars) is given by \( C(x, y) = 5x^2 + 2xy + 3y^2 + 800 \).

(a). If the total number of items (both types combined) is 39, find the minimum cost \( C(x, y) \).

(b). Estimates the additional minimal cost if the total production quota is raised to 41.
11 (12 pts). Find $x, y, z$ such that
\[
\begin{bmatrix} \begin{array}{cc} x + y & 2 \\ 4 & 0 \end{array} \end{bmatrix} = \begin{bmatrix} \begin{array}{cc} 6 & x - y \\ 4 & z \end{array} \end{bmatrix}.
\]

12 (13 pts). Let 
\[
A = \begin{bmatrix} 2 & -3 & 4 \\ 0 & 2 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 & 0 \\ 5 & 1 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} -3 & 0 & 5 \\ 2 & 1 & 3 \end{bmatrix}.
\]
Find $2A - 5(B + C)$.

13 (12 pts). Let matrix 
\[
A = \begin{bmatrix} a & 1 - a \\ 1 + a & -a \end{bmatrix},
\]
find $A^2$ (which is the matrix $A \cdot A$).
14 (13 pts). Solve following system. If the system has infinitely many solution, formulate them using parameter(s). Show your work indetails.

\[
\begin{cases}
    x + 2y + z = 1 \\
    2x - y + z = 2 \\
    3x + y + 2z = 3
\end{cases}
\]

15 (13pts). It is given that the inverse \[
\begin{pmatrix}
    -1 & 1 & 1 \\
    16 & -14 & -13 \\
    6 & -5 & -5
\end{pmatrix}
\]^{-1} = \[
\begin{pmatrix}
    5 & 0 & 1 \\
    2 & -1 & 3 \\
    4 & 1 & -2
\end{pmatrix}
\]. Make use of the above given information to solve the linear system (write out your formulation):

\[
\begin{cases}
    -x + y + z = 6 \\
    16x - 14y - 13z = 2 \\
    6x - 5y - 5z = 7
\end{cases}
\]
Choose One of the following problems. Do both get extra credits.

16 (12pts). A closed economy system consists of three sectors $A, B, C$. The input–output matrix is given by

\[
\begin{array}{ccc}
  & A & B & C \\
 A & 0.2 & 0.1 & 0.2 \\
 B & 0.2 & 0.3 & 0.4 \\
 C & 0.6 & 0.6 & 0.4 \\
\end{array}
\]

Find the salaries $X = [x, y, z]$ for sectors $A, B$ and $C$. (You may set the $z = 30,000$.)

17 (12 pts). The revenue and cost functions for a company are given below in the figure. Should the company produce the 500th item in addition to a total production of $q = 499$? Show your argument in detail.