Below you will find 10 problems, each worth 10 points. Solve the problems in the space provided. When writing a solution to a problem, show all work. No books or notes are allowed. Sign and submit your formula sheet with the exam.

**Problem 1.** Convert the units as indicated:

(a) $12.35^\circ$ to degrees, minutes and seconds.

(b) $\frac{7\pi}{72}$ (radians) to degrees, minutes and seconds.
**Problem 2.** Find two positive coterminal angles and one negative coterminal angle for each of the following angles:

(a) \(-230^\circ\) (use degrees);

(b) \(\frac{9\pi}{11}\) (use radians).

**Problem 3.** An angle \(\theta\), in standard position, is located in the third quadrant and has \(\cot \theta = \frac{20}{21}\). Find the exact values of \(\sin \theta\) and \(\cos \theta\).

**Problem 4.** Find the exact values of \(\sin \left(-\frac{5\pi}{4}\right)\) and \(\cos \left(-\frac{5\pi}{4}\right)\).
Problem 5. Find the length of the arc that subtends the angle $100^\circ$ on a circle of diameter 18 in.

Problem 6. Prove the identity: $\sin^2 t (\csc^2 t - 1) = \cos^2 t$.

Problem 7. Find the exact values of $t$, in the interval $[-\pi, 5\pi]$, which satisfy the equation

$$\sin t = -\frac{\sqrt{2}}{2}.$$
Problem 8. Find the equation of the line that passes through the points $A(-2, 2)$ and $B(3, -8)$.

Problem 9. Find the equation of the line that passes through the point $P(2, -1)$ and is parallel to the line $3x - 4y = 7$.

Problem 10. Find the equation of the line that passes through the point $Q(-1, 2)$ and is perpendicular to the line $x - 4y = 12$. 
PLANE TRIGONOMETRY

Exam II
October 11, 2007

Below you will find 10 problems, each worth 10 points. Solve the problems in the space provided. When writing a solution to a problem, show all work. No books or notes are allowed. Sign and submit your formula sheet with the exam.

Problem 1. A rocket, flying on a straight vertical trajectory, is observed from a point positioned 5000 meters away from the launch site. During the final minute of flight, the angle of elevation of the rocket (measured from the observation point) changed from $32^\circ$ to $61^\circ$. How long was the flight path of the rocket during this period? Approximate your answer to the nearest tenth of a meter.
**Problem 2.** The graph of the equation \( y = a \sin(bx + c) \), with \( a > 0 \) and \( b > 0 \), is shown in the figure below.

(a) Find the amplitude and the period.

(b) Find the coefficients \( a \), \( b \) and \( c \).

![Graph of \( y = a \sin(bx + c) \)](image-url)

**Problem 3.** Find the amplitude, period, and phase shift for the curve \( y = 3 \sin \left( 4x + \frac{\pi}{3} \right) \).

**Problem 4.** An angle \( \theta \), in standard position, has its terminal side in Quadrant IV on the line \( 12x + 5y = 0 \). Find the **exact** values of \( \sin \theta \) and \( \cos \theta \).
Problem 5. Find all solutions of the equation: \( \sin \left( 4x - \frac{\pi}{3} \right) = 1 \). Use exact values.

Problem 6. Find exact values of \( \sin(-600^\circ) \) and \( \cos(-600^\circ) \), using the following procedure: (1) find the quadrant; (2) find the reference angle \( \theta_R \); (3) use previous steps and the reference angle formulas \( \sin \theta = \pm \sin \theta_R \) and \( \cos \theta = \pm \cos \theta_R \).

Problem 7. Find the solutions of the equation \( 4 \cos^2 t + 4 \cos t - 3 = 0 \), that are in the interval \([0, 2\pi)\). Use exact values
Problem 8. Verify the identity: \( \sin x + \cos x \cot x = \csc x \).

Problem 9. Given the right triangle \( \triangle ABC \), with \( \gamma = 90^\circ \), \( a = 3 \text{ cm} \), and \( b = 7 \text{ cm} \), find the remaining elements of the triangle: the side \( c \), and the angles \( \alpha \) and \( \beta \). (When computing the angles, express them in degrees, rounded to the nearest tenth.)

Problem 9. Find the sides labeled \( x \) and \( y \) in the right triangle:
Below you will find 10 problems, each worth 10 points. Solve the problems in the space provided. When writing a solution to a problem, show all work. No books or notes are allowed. Sign and submit your formula sheet with the exam.

**Problem 1.** The triangle $\triangle ABC$ has $a = 5 \text{ cm}$, $b = 6 \text{ cm}$, and $\hat{A} = 30^\circ$. Find the remaining parts: $\hat{B}$, $\hat{C}$, and $c$. (Express all angles in degrees. Round to two decimal places.)

**Warning:** This problem has two solutions!
Problem 2. The triangle $\triangle ABC$ has $a = 10\text{ cm}$, $\hat{B} = 45^\circ$, and $\hat{C} = 60^\circ$. Find the remaining parts: $\hat{A}$, $b$, and $c$. (Express all angles in degrees. Round to two decimal places.)

Problem 3. The triangle $\triangle ABC$ has $a = 15\text{ cm}$, $b = 4\text{ cm}$ and $\hat{C} = 60^\circ$. Find the remaining parts: $\hat{A}$, $\hat{B}$, and $c$. (Express all angles in degrees. Round to two decimal places.)

Problem 4. Find the exact value of $\cot \left[ \arcsin \left( -\frac{1}{3} \right) \right]$. 
Problem 5. Given $\alpha$ and $\beta$ in Quadrant III, with $\tan \alpha = \frac{4}{3}$ and $\cos \beta = -\frac{12}{13}$, find the exact values of $\tan(\alpha + \beta)$ and $\tan(\alpha - \beta)$.

Problem 6. Given $\theta$ in Quadrant II, with $\sin \theta = \frac{8}{17}$, find the exact values of $\sin(2\theta)$ and $\cos(2\theta)$.

Problem 7. Suppose $180^\circ \leq \theta \leq 360^\circ$, and $\cos \theta = -\frac{119}{169}$. Find the exact values of $\sin(\theta/2)$ and $\cos(\theta/2)$. 
Problem 8. The triangle $\triangle ABC$ has $a = 5$ cm, $b = 6$ cm, and $c = 7$ cm. Find the remaining parts: $\hat{A}$, $\hat{B}$, and $\hat{C}$. (Express all angles in degrees. Round to two decimal places.)

Problem 9. Verify the identity: $\frac{\sin(4\theta) + \sin(6\theta)}{\cos(4\theta) - \cos(6\theta)} = \cot \theta$.

Problem 10. Find all solutions of the equation $6\cos^2 t + 5\cos t - 6 = 0$, that are in the interval $[0, 2\pi)$. (Use radians and round to two decimal places.)
**PLANE TRIGONOMETRY**

**Final Exam**

December 14, 2007

<table>
<thead>
<tr>
<th>Page 1</th>
<th>Page 2</th>
<th>Page 3</th>
<th>Page 4</th>
<th>Page 5</th>
<th>Page 6</th>
<th>Page 7</th>
<th>TOTAL</th>
</tr>
</thead>
<tbody>
<tr>
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Below you will find 20 problems, each worth 10 points. Solve the problems in the space provided. When writing a solution to a problem, **show all work**. No books or notes are allowed. **Sign and submit your formula sheet with the exam.**

**Problem 1.** Let \( u \) be an angle in quadrant II, with \( \tan u = -\frac{12}{5} \). Find the **exact** values of \( \sin u \) and \( \cos u \).

**Problem 2.** Verify the identity: \( \frac{\cot^2 x}{\csc x + 1} = \frac{1 - \sin x}{\sin x} \).
Problem 3. Let $\theta$ be an angle (in standard position) in quadrant IV, whose terminal side is parallel to the line $3x + 4y = 12$. Find the exact values of $\sin \theta$ and $\cos \theta$.

Problem 4. Let $\alpha$ and $\beta$ be two angles in quadrant II, with $\sec \alpha = -\frac{5}{4}$ and $\sin \beta = \frac{15}{17}$. Find the exact values of $\sin(\alpha + \beta)$ and $\cos(\alpha + \beta)$.

Problem 5. Find all solutions of the equation $2 \cos^2 x - 7 \cos x + 3 = 0$, that are in the interval $[0, 2\pi]$. Use exact values.
Problem 6. Given $180^\circ \leq \theta \leq 360^\circ$, with \( \cos\theta = -\frac{7}{25} \). Find the \textbf{exact} values of \( \sin(\theta/2) \) and \( \cos(\theta/2) \).

Problem 7. Find all solutions of the equation: \( \sin 5t - \sin t = 0 \). Use \textbf{exact} values.

Problem 8. Solve \( \triangle ABC \), given: \( a = 5, b = 6, \hat{A} = 30^\circ \). (Express all angles in degrees. Round to two decimal places.)
Problem 9. Solve \( \triangle ABC \), given: \( a = 7,\ b = 2,\ \hat{C} = 60^\circ \). (Express all angles in degrees. Round to two decimal places.)

Problem 10. Solve \( \triangle ABC \), given: \( a = 5,\ b = 7,\ c = 9 \). (Express all angles in degrees. Round to two decimal places.)

Problem 11. Express the complex number \( z = 2 - 2i \) in trigonometric form:

\[
 z = r (\cos \theta + i \sin \theta) \quad (\text{or} \quad z = \text{cis} \theta),
\]

with \( 0 \leq \theta < 2\pi \).
Problem 12. Find the equation of the line that passes through the point $P(-1, 2)$, and is perpendicular to the line $x + 5y = 10$.

Problem 13. Find the vertex, focus, and directrix, for the parabola $-8(x + 2) = (y - 5)^2$.

Problem 14. Find the equation of the parabola with vertex $V(1, 2)$ and focus $F(1, -2)$. 
Problem 15. Find the center, vertices, and foci, for the ellipse:

\[ 16(x - 2)^2 + 9y^2 = 144. \]

Problem 16. Find the equation of the ellipse with vertices \( V(\pm4, -1) \) and foci \( F(\pm2, -1) \).

Problem 17. Find the center, vertices, and the asymptotes, for the hyperbola:

\[ \frac{(y + 1)^2}{4} - \frac{(x - 3)^2}{9} = 1. \]
Problem 18. Find the equation of the hyperbola with vertices $V(-1, \pm 1)$ and foci $F(-1, \pm 5)$.

Problem 19. Find the polar equation that has same graph as $(x + 2)^2 + (y - 1)^2 = 5$. The polar equation should have the form: “$r = \text{expression in } \theta$.”

Problem 20. Let $L$ be a line, with polar equation $r = \frac{12}{2 \cos \theta - 3 \sin \theta}$.

(a) Find the equation of $L$ in rectangular coordinates $(x, y)$.

(b) Find the $x$-intercept and the $y$-intercept of $L$. 