1. (16 points) Perform the indicated divisions:
   (a) \((x^3 + 8x^2 + 5x - 3) \div (x - 1)\). Quotient: \(\frac{x^2 + 9x + 14}{x - 1}\), Remainder: 11.
   \[
   \begin{array}{c|cccc}
   & x^3 & + 8x^2 & + 5x & - 3 \\
   \hline
   x - 1 & x^2 & + 9x & + 14 & \\
   \hline
   & -x^2 & - x^2 & \\
   & & -x^2 & - x & - 3 \\
   & & & 9x & + 9x & - 9x \\
   & & & -9x & - 9x & - 11 \\
   & & & & 11 & \\
   \end{array}
   \]

   (b) \((3x^4 - 1) \div (x^2 + x + 4)\). Quotient: \(\frac{3x^2 - 3x - 9}{x^2 + x + 4}\), Remainder: \(11x + 35\).
   \[
   \begin{array}{c|c c c c c}
   & 3x^4 & + 0x^3 & + 0x^2 & + 0x & - 1 \\
   \hline
   x^2 + x + 4 & 3x^2 & - 3x & - 9 & \\
   \hline
   & -3x^4 & - 3x^3 & - 12x^2 & \\
   & & -3x^3 & - 3x^2 & - 12x \\
   & & & -9x^2 & - 9x & - 27 \\
   & & & & 11x & + 35 \\
   \end{array}
   \]

2. (7 points) \(f(x) = 6x^4 - 19x^3 + 29x^2 - 15x - 9\).
   (a) Use the rational zeros test to list all the possible rational zeros of the polynomial \(f\).
   \[
   \frac{a}{b} \text{ divides } \frac{9}{b} \Rightarrow a = \pm 1, \pm 3, \pm 9 \Rightarrow \frac{a}{b} \in \left\{ \pm 1, \pm \frac{9}{3}, \pm \frac{3}{3}, \pm \frac{3}{3}, \pm \frac{2}{3}, \pm \frac{1}{3}, \pm \frac{1}{3} \right\}
   \]
   (b) What are the actual rational zeros of \(f\)? You may use your calculator to further reduce the possibilities you listed in (a).
   \[
   f(-\frac{1}{3}) = 0, \quad f(3) = 0
   \]

3. (6 points) Find the zeros of the polynomial \(p(x) = x^6 + 2x^5 - 8x^4\) and their multiplicities.
   \[
   p(x) = x^4(x^2 + 2x - 8) = x^4(x + 4)(x - 2)
   \]
   Zeros: \(x = 0\) multiplicity 4, \(x = -4\) multiplicity 1, \(x = 2\) multiplicity 1.
4. (7 points) Solve the polynomial inequality \(6x^4 \leq 3x^3\). Give the solution set in interval notation. (If you use your calculator, explain what you are doing.)

For \(x < 0\) we have \(6x^4 > 0\) and \(3x^3 \leq 0\) so \(6x^4 > 3x^3\).

Thus our inequality has no negative solutions. For \(x = 0\), \(6x^4 = 0\) and \(3x^3 = 0\), so \(x = 0\) is a solution. For \(x > 0\) we have \(3x^3 > 0\) so \(6x^4 \leq 3x^3 \iff \frac{6x^4}{3x^3} \leq \frac{3x^3}{3x^3} \iff 2x \leq 1 \iff x \leq \frac{1}{2}\).

Thus the solution set is \([0, \frac{1}{2}]\).

5. (9 points) Solve the rational inequality \(\frac{4}{x} < \frac{5}{x+1}\) and sketch the solution set on the real number line below. Remember to indicate the status of endpoints.

\[\frac{4}{x} < \frac{5}{x+1}\]

is equivalent to \(0 < \frac{5}{x+1} - \frac{4}{x}\). Thus the related function is \(f(x) = \frac{5}{x+1} - \frac{4}{x} = \frac{5x - 4(x+1)}{x(x+1)} = \frac{x - 4}{x(x+1)}\).

Note that \(x = 0\) and \(x = -1\) are not allowed in our inequality. Moreover, \(f(x) = 0 \iff x = 4\), so the critical points are \(-1, 0, 4\).

<table>
<thead>
<tr>
<th>Interval</th>
<th>((-\infty, -1))</th>
<th>((-1, 0))</th>
<th>((0, 4))</th>
<th>((4, \infty))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Test value</td>
<td>(f(-2) = -3)</td>
<td>(f\left(-\frac{1}{2}\right) = 18)</td>
<td>(f(1) = -1.5)</td>
<td>(f(5) = 0.03)</td>
</tr>
<tr>
<td>Sign of (f(x))</td>
<td>(-)</td>
<td>(+)</td>
<td>(-)</td>
<td>(+)</td>
</tr>
</tbody>
</table>

Solution set for \(0 < f(x)\) is \((-1, 0) \cup (4, \infty)\).

6. (10 points) (a) If \(f(x) = x^2 - x + 2\) and \(g(x) = x - 3\), then the composition \((f \circ g)(x) = x^2 - 7x + 14\),

\[(f \circ g)(x) = f(g(x)) = f(x - 3) = (x - 3)^2 - (x - 3) + 2 = x^2 - 6x + 9 - x + 3 + 2\]
\[= x^2 - 7x + 4\]

(b) If \(h(x) = 2x^3 - 7\), then the inverse function \(h^{-1}(x) = \frac{\sqrt[3]{x + 7}}{2}\).

Write \(y = 2x^3 - 7\). Interchange \(x\) and \(y\) to get \(x = 2y^3 - 7\).

Solve for \(y\): \(\frac{x + 7}{2} = y^3\) so \(y = \sqrt[3]{\frac{x + 7}{2}}\). Replace \(y\) by \(h^{-1}(x)\).

7. (2 points) The graph of a polynomial \(f\) has 7 turning points. The smallest degree \(f\) can have is 8.

8. (5 points) Express in terms of natural logarithms \(\log_4 17 = \frac{\ln 17}{\ln 4}\) (Use Change-of-base Formula).
9. (18 points) Provide the following information and use it to graph the rational function

\[ g(x) = \frac{x^2 - 9}{x^2 - 16}. \]

(a) The y-intercept: \( g(0) = \frac{-9}{16} = -\frac{9}{16} \) so y-intercept \( (0, -\frac{9}{16}) \)

(b) The x-intercept(s):
\[ x^2 - 9 = (x - 3)(x + 3) = 0 \iff x = \pm 3 \]
so x-intercepts are \( (3, 0) \) and \( (-3, 0) \)

(c) Equations of any vertical asymptotes:
\[ x^2 - 16 = (x - 4)(x + 4) = 0 \iff x = \pm 4 \]
(Note \( x^2 - 9 = 0 \) for \( x = \pm 4 \),)

(d) Equation of any horizontal asymptote:
Degree of \( x^2 - 9 = x^2 - 16 \) so \( g \) has
horizontal asymptote \( y = \frac{1}{4} \)

(e) Symmetry: Is \( g(x) \) even, odd or neither?
\[ g(-x) = \frac{(-x)^2 - 9}{(-x)^2 - 16} = \frac{x^2 - 9}{x^2 - 16} = g(x) \] so \( g \) is even

All asymptotes dotted.

10. (12 points) For the following rational functions find the vertical, horizontal and oblique asymptotes (write an equation for the asymptote in the space provided or N/A if there is not an asymptote of that type):

<table>
<thead>
<tr>
<th>( f(x) = \frac{x + 3}{2x^2 + 1} )</th>
<th>vertical</th>
<th>horizontal</th>
<th>oblique</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x+3 \approx 0 ) for all real ( x )</td>
<td>( x+3 \approx 0 ) for all real ( x )</td>
<td>( y = 0 )</td>
<td></td>
</tr>
<tr>
<td>( N/A )</td>
<td>( N/A )</td>
<td>( N/A )</td>
<td></td>
</tr>
</tbody>
</table>

Above \( \text{Deg}(x+3) \) means degree of polynomial \( x+3 \) etc.

10. (4 points) The polynomial \( x^4 - 6x^3 + 33x^2 - 102x - 26 \) has zeros \( 1 - 5i \) and \( 2 + \sqrt{5} \). The other zeros are \( \frac{1+5i}{2-\sqrt{5}} \). (Hint: No calculation is required.)

see Section 3.3, p. 217 in Text Book.

12. (4 points) Use the leading-term test to determine which graph best represents the behavior of the given polynomial as \( x \to \infty \) and \( x \to -\infty \): (Use Leading Term Test Sect. 3.1, p. 197.)

(a) \(-2x^6 + 5000x^4 + 3\). Circle One: A B C D. (b) \(-3000x^2 + x^3 + 700\). Circle One: A B C D.

Leading Term \( = -2x^6 \)

Even degree: Negative Leading Coeff.

Leading Term \( = x^3 \)

Odd degree: Positive Leading Coeff.