MATH 100  College Algebra – Exam III
7:15–8:15pm, Tuesday November 1, 2005

Check that you have all three pages. SHOW ALL YOUR WORK. Write complex answers in $a + bi$ form.

1. (16 points) Perform the indicated divisions:
   (a) $(x^3 + 3x^2 - 10x - 7) \div (x - 3)$. Quotient=______________, Remainder=______.

   (b) $(2x^4 - 9) \div (x^2 - x + 3)$. Quotient=______________, Remainder=______.

2. (7 points) $f(x) = 10x^4 + 9x^3 + 6x^2 - 4x - 3$.
   (a) Use the rational zeros test to list all the possible rational zeros of $f$.

   (b) What are the actual rational zeros of $f$? You may use your calculator to reduce the possibilities.

3. (7 points) Find the zeros of $p(x) = x^7 + 8x^6 + 16x^5$ and their multiplicities.
4. (14 points) The polynomial \( h(x) = x^3 + x^2 - x + 15 \) has \( h(-3) = 0 \).

(a) A linear factor of \( h(x) \) is \_____.

(b) Divide \( h(x) \) by the factor in (a) to find the remaining quadratic factor.

(c) Find the zeros of \( h(x) \).

(d) Factor \( h(x) \) into three linear factors (using complex numbers if necessary).

5. (7 points) Solve the inequality \( 2x - 6 > 7(x - 1) + 3 \). Give the solution set in interval notation.

6. (9 points) Solve the inequality \( |3x - 5| > 4 \) and sketch the solution set on the real number line below. Remember to indicate the status of endpoints.

7. (2 points) The graph of a polynomial \( f \) has 6 turning points. The smallest degree \( f \) can have is ____.
8. (18 points) Provide the following information and use it to graph

\[ g(x) = \frac{2(25 - x^2)}{x^2 - 9}. \]

(a) The y-intercept:

(b) The x-intercept(s):

(c) Equations of any vertical asymptotes:

(d) Equation of any horizontal asymptote:

(e) Symmetry: Is \( g(x) \) even, odd or neither?

9. (12 points) For the following functions find the vertical, horizontal and oblique asymptotes (write an equation for the asymptote in the space provided or N/A if there is not an asymptote of that type):

<table>
<thead>
<tr>
<th></th>
<th>vertical</th>
<th>horizontal</th>
<th>oblique</th>
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</thead>
<tbody>
<tr>
<td>( f(x) = \frac{x + 4}{x^2 + 16} )</td>
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<td></td>
<td></td>
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<tr>
<td>( f(x) = \frac{x^2 - 3x}{x + 5} )</td>
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10. (4 points) The polynomial \( x^4 - 4x^3 + 5x^2 - 46x + 70 \) has zeros \(-1 + 3i\) and \(3 + \sqrt{2}\). The other zeros are _________, _________.

11. (4 points) Use the leading-term test to determine which graph best represents the behavior of the given polynomial as \( x \to \infty \) and \( x \to -\infty \):

(a) \(-3x^4 + 3000x^3 + 7\). Circle One: A B C D.

(b) \(-1000x^4 + 2x^5 + 500\). Circle One: A B C D.