MATH 100  College Algebra – Exam I
7:15–8:15pm, Tuesday February 7, 2006

Check that you have all three pages. SHOW ALL YOUR WORK.

1. (6 points) Simplify \( \frac{(x^{-1}y^3)^2}{(yx^{-3})^3} \) avoiding negative exponents (you may assume that \( x, y \neq 0 \)):

2. (8 points) Write as a rational power of \( a \) (assume \( a \) is positive):
   (i) \( \sqrt[3]{\frac{\sqrt{a^2}}{\sqrt[3]{a}}} \)
   (ii) \( \sqrt[7]{a^2 \sqrt{a}} \)

3. (7 points) Simplify by extracting squares (\( y \) could be negative in (a)):
   (a) \( \sqrt{y^8} = \)
   (b) \( 7\sqrt{28} - 3\sqrt{63} = \)

4. (5 points) Multiply out and write the polynomial in standard form: \((2x^2 - x + 7)(2x^2 + x + 7)\).

5. (6 points) Simplify: \( \frac{x^2 - 5x - 14}{2x^2 - 14x} \).
6. (12 points) Factor the following polynomials as far as possible:
(a) \(3x^2 + x - 2 = \)

(b) \(25x^2 - 16 = \)

(c) \(4(x - 6)^3 + (x + 1)(x - 6)^2 = \)

7. (7 points) Simplify: \(\frac{5}{a+2} - \frac{4}{a+1} \).

8. (7 points) Perform the addition and simplify: \(\frac{4}{z^2 - 9} + \frac{2}{z^2 - 9z + 18} \).

9. (8 points) Use your calculator to graph 
\[y = x^3 - x^2 - 6x + 1,\]
with the window settings shown.
Find the coordinates of any intercepts correct to three decimal places:
\[x\text{-intercepts} = \boxed{\text{_________________}} \]
\[y\text{-intercept} = \boxed{\text{______________}} \]
10. (9 points) The points (3, −1) and (−1, 1) are endpoints of the diameter of a circle.
   (a) What is the length of the diameter?

   (b) What is the center of the circle?

   (c) Write down an equation of the circle.

11. (5 points) Rationalize the numerator: \( \frac{8 + \sqrt{3}}{6 - \sqrt{3}} \)

12. (6 points) (a) The slope of the line passing through (−2, 8) and (4, 2) is \( m = \) _____.

   (b) Find an equation for the line passing through the two points in part (a).

13. (6 points) (a) The slope of the line \( 2x + 5y = 1 \) is \( m_1 = \) _____.

   (b) The slope of a line perpendicular to the line in (a) is \( m_2 = \) _____.

14. (4 points) What is the domain of the function \( f(x) = \frac{x^2 + x}{x^2 + 5x - 6} \)?

15. (4 points) Let \( g(y) = y^2 + 3y - 2 \). Find and simplify \( g(y - 4) \).