MATH 100 College Algebra – Final Exam
7:00–8:50pm, Wednesday December 14, 2005

Check that you have all five pages. SHOW ALL YOUR WORK. Write complex answers in $a + bi$ form.

1. (6 points) Perform the division: $(3x^3 - 8x^2 - 7) \div (x - 3)$:
   Quotient=______________, Remainder=______.

2. (12 points) $f(x) = x^3 + 2x^2 - 12x - 16$.
   (a) Use the rational-zeros test to find a rational zero of $f$. List all the possible rational zeros given by the test, as well as the actual zeros.
   (b) Find the remaining zeros of $f$ (including complex zeros).

3. (10 points) Factor into linear factors (using complex numbers where necessary):
   (a) $5x^2 + 7x - 6 =$
   (b) $x^4 - x^2 - 2 =$

4. (10 points) (a) Find the slope of the line passing through $(3, -1)$ and $(-1, 7)$.
   (b) An equation for the line in (a) is $y =$ ________________.
5. (8 points) (a) Find the critical values for the inequality $x^3 \leq 4x$.

(b) Solve the inequality in (a). Sketch the solution below (remember to indicate the status of endpoints).

6. (10 points) Find all $x$ satisfying the equation: $\frac{2x}{x-3} - \frac{18}{x^2 - 3x} = \frac{5}{x}$.

7. (12 points) Provide the following information and use it to graph $g(x) = \frac{3x^2}{x^2 - 16}$.

(a) The $x$-intercept(s) and $y$-intercept(s):

(b) Equations for all the vertical asymptotes:

(c) Equation of the horizontal asymptote:

(d) Symmetry: Is $g(x)$ even, odd or neither?

8. (12 points) For the following functions find the vertical, horizontal and oblique asymptotes (write an equation for the asymptote in the space provided or N/A if there is not an asymptote of that type):

<table>
<thead>
<tr>
<th>$f(x)$</th>
<th>vertical</th>
<th>horizontal</th>
<th>oblique</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(x) = \frac{2x^2 + 3x}{x - 3}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$f(x) = \frac{5x - 10}{x^2 - 5x}$</td>
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</tbody>
</table>
9. (9 points) Solve the inequality \(|2x - 7| \leq 5\). Give your answer in interval notation.

10. (10 points) Find all the real numbers \(x\) which satisfy the equation \(\sqrt{x+2} + 4 = x\).

11. (12 points) (a) Simplify: \[
\frac{2}{x+1} - \frac{1}{4} - \frac{x-1}{3} - \frac{3}{x}.
\]

(b) Perform the subtraction and simplify: \[
\frac{4}{y^2 - 4} - \frac{5}{y^2 + y - 6}.
\]

12. (10 points) Simplify so that only positive rational exponents occur (assume that \(x\) and \(y\) are positive):

(a) \[
\frac{(x^2y^2)^2}{x^2y^3}
\]

(b) \[
\sqrt[5]{y\sqrt{y^3}}
\]
13. (8 points) (a) Find the critical values for the inequality \(1 \leq \frac{3}{x + 1}\).

(b) Solve the inequality in (a). Sketch the solution below (remember to indicate the status of endpoints).

14. (9 points) The graph of \(y = f(x)\) is shown in the first picture. Sketch the graphs requested on the other two. Underneath describe the transformations in words.

15. (16 points) (a) Write as a single logarithm: \(\frac{1}{3}\ln x - 3\ln y + 2\ln z = \)

(b) Express in terms of natural logarithms: \(\log_5 19 = \)

(c) Solve the logarithmic equation \(\log_2(x + 5) - \log_2(x) = 3\).

16. (8 points) Use the method of elimination or substitution to solve the system of equations

\[
\begin{align*}
7x + 5y &= 4 \\
3x + 2y &= 1.
\end{align*}
\]
17. (12 points) (a) If \( f(x) = x^2 + x + 3 \) and \( g(x) = x - 1 \), then the composition \((f \circ g)(x) = \)

(b) If \( h(x) = \sqrt[3]{3x - 2} \), then the inverse function \( h^{-1}(x) = \)

18. (10 points) A savings account offers an annual rate of 3.25% compounded continuously.
(a) If $10,000 is deposited in the account what will be the balance after 5 years?

(b) What is the tripling time (i.e., the time for the amount in the account to triple)?

19. (8 points) (a) Give the augmented matrix for the system of equations:

\[
\begin{align*}
3x - 2y + 5z &= 8 \\
2x + y - 3z &= 1 \\
x - 3y - 2z &= 2.
\end{align*}
\]

(b) Use your calculator to find the reduced row-echelon form of the matrix in (a).

(c) Use (b) to solve the system of equations in (a): \( x = \) ____, \( y = \) ____, \( z = \) ____.  

20. (8 points) (a) Write the system as a matrix equation \( AX = B \)

\[
\begin{align*}
2x + y - 3z &= 2 \\
x - y + z &= 5 \\
x - z &= 1.
\end{align*}
\]

(b) Use your calculator to find \( A^{-1} \), the inverse of the coefficient matrix \( A \), then \( A^{-1}B \).

\[
A^{-1} = \quad \quad \quad A^{-1}B =
\]

(c) Use (b) to solve the system of equations in (a): \( x = \) ____, \( y = \) ____, \( z = \) ____.