Representations of Algebraic Groups, Quantum Groups, and Lie Algebras

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Contents

Cohomology of line bundles
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This will be a survey of some of the known results on cohomology of line bundles on flag varieties and their quantized analogues. At the same time we shall call attention to several open questions concerning these. Moreover, we shall show how a certain related problem is connected with line bundle cohomology on the corresponding cotangent bundle where the situation is wide open even in characteristic zero.

Representation type of Hecke algebras and the Poincare polynomial
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In this talk, I will explain my result how the Poincare polynomial of the Weyl group controls the representation type of the corresponding Hecke algebra.

Cohomology for Frobenius kernels
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Let G be a simple algebraic group over an algebraically closed field k of characteristic $p > 0$ and G_r denote the rth Frobenius kernel of G. For p larger than the Coxeter number, an elegant formula was found by Andersen and Jantzen for the G_1-cohomology of standard induced modules in all degrees. This talk will present recent computations of first and second cohomology groups for small primes and higher Frobenius kernels. For G_1, the generic answer in fact holds for most primes. The computations for G_r are made by computing B_r-cohomology groups of simple B-modules for a Borel subgroup B of G. Also used are computations of ordinary Lie algebra cohomology of the Lie algebra of the unipotent radical of B for small primes.

Varieties of Nilpotent Matrices for Simple Lie Algebras: Restricted Nullcones and Support Varieties

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In this talk I will survey recent results on determining subvarieties of the nullcone of a simple Lie algebra $\mathfrak{g}$. One example of such a variety is the restricted nullcone, which arises as the spectrum of the cohomology ring for the restricted Lie algebra cohomology of $\mathfrak{g}$. We have completed the determination of the restricted nullcone for all primes. Other examples are the support varieties of Weyl modules. I will discuss recent progress in computing these support varieties over arbitrary characteristic.

Endotrivial modules for finite groups of Lie type

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This is a report on joint work with Nadia Mazza and Dan Nakano. Let G be a finite group and k a field of characteristic $p > 0$. A kG-module is and endotrivial module provided $\hom_k(M, M) \cong k \oplus P$ where P is a projective module. Among other things, the endotrivial modules form a part of the Picard group of self-equivalences of the stable category of kG-modules. Recently, the first author and Jacques Thévenaz completed a classification of the endotrivial modules over p-groups. Building on this work the authors obtain a classification of the endotrivial modules over groups finite of Lie type including the twisted types. A major part of the project is determining which endotrivial modules for the Borel have endotrivial Green correspondents. We show that Alperin's formula for the torsion-free rank of the group of endotrivial modules holds even when the group is not a p-group.

Derived equivalences between blocks of $GL(n)$
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Raphael Rouquier and I completed a proof of Broue's abelian defect group conjecture for symmetric groups using derived equivalences arising from '$\mathfrak{sl}(2)$-categorifications'. I will describe analogous derived equivalences between blocks of rational representations of $GL(n)$.

Generators and relations for generalized q-Schur algebras
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Generalized Schur algebras were defined by Donkin by truncating the representation theory of an algebraic group (or its hyperalgebra) at a finite ideal in the dominant weight poset. These finite-dimensional algebras provide a broad class of quasihereditary algebras closely connected with fundamental problems in Lie theory and finite groups.

In this talk I will outline a presentation for such algebras (and their quantizations) via generators and relations, inspired by Lusztig's "modified form" of a quantized enveloping algebra. This extends earlier work from type A to arbitrary (finite) type. As a result the generalized Schur algebras inherit a canonical basis. This yields in particular an easy new proof of the (known) fact that these algebras are quasihereditary.

Strong monomial basis property and canonical basis for a cyclic quiver
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This is a report based on a series of papers with Bangming Deng (and Jie Xiao). We investigate the correspondence between the isomorphism classes of nilpotent representations of a cyclic quiver and the orbits in the corresponding representation varieties. Using generic extensions we endow the set $\mathcal{M}$ of such isoclasses with a monoid structure and identify the submonoid $\mathcal{M}_c$ generated by simple modules. On the other hand, we use the partial ordering on the orbits (i.e., the Bruhat-Chevalley type ordering) to induce a poset structure on $\mathcal{M}$ and describe the poset ideal generated by an element of the submonoid $\mathcal{M}_c$ in terms of the existence of a certain composition series of the corresponding module. As applications of these results, we obtain a systematic description of many monomial bases for any given quantum affine $\mathfrak{sl}_n$. We then use this strong monomial basis property to introduce a new integral PBW-like basis for the Lusztig $\mathbb{Z}[v, v^{-1}]$-form of $\mathbf{U}^+$ and further to present an elementary algebraic construction of the canonical bases for both the Hall algebra of a cyclic quiver and the $+$-part $\mathbf{U}^+$ of the quantum affine $\mathfrak{sl}_n$.

π-points for finite group schemes

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I shall present joint work with Julia Pevtsova in which we provide a generalization to all finite group schemes over a field k of characteristic $p > 0$ of "cyclic shifted subgroups" for elementary abelian p-groups. The space $\Pi(G)$ of π-points is homeomorphic to the homogeneous prime ideal spectrum of $H^*(G, k)$, and leads to the useful invariant $M \mapsto \Pi(G)_M$ for arbitrary G-modules. As an application, we determine the thick, tensor-ideal subcategories of $stmod(G)$.

Fixed point functors for symmetric groups and Schur algebras.

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We will present work on the representation theory of the symmetric group and Schur algebra. We prove that for partitions with first row less than p, first row removal induces an injection on the Ext^1 space between the corresponding irreducibles. We conjecture this to hold for arbitrary partitions. If true this conjecture would imply the Kleshchev-Martin conjecture than in characteristic > 2 there are no self-extensions between simple modules. We also discuss an interesting fixed-point functor that arises in the proof.

Nilpotent Orbits in Restricted Symmetric Spaces

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Let G be a reductive algebraic group over an algebraically closed field of characteristic p not 2, and let $\theta \in$
$operatornameAut(G)$ be an involution with fixed-point subgroup $K = G^\theta$. The Lie algebra $\mathfrak{g} = \mathrm{Lie}(G)$ of G decomposes as a direct sum $\mathfrak{g} = \mathfrak{k} \oplus \mathfrak{p}$, where $\mathfrak{k} = \mathrm{Lie}(K)$ and $\mathfrak{p}$ identifies with the tangent space at the identity to the symmetric space G/K. The algebraic group K acts on $\mathfrak{p}$, and when $p > 2$, the restricted Lie algebra structure on $\mathfrak{g}$ induces a restricted Lie triple system structure on the infinitesimal symmetric space $\mathfrak{p}$.

When $k = \mathbb{C}$, the K-orbits on the nullcone $\mathcal{N}(\mathfrak{p}) = \mathcal{N}(\mathfrak{g}) \cap \mathfrak{p}$ have been classified (e.g., when $\mathfrak{g}$ is semisimple, and G is its adjoint group). They figure prominently in a number of settings, due to the associated classification of real nilpotent orbits of a real Lie group via a bijection of Sekiguchi. Applications include the study of the associated varieties of Harish-Chandra modules and the coadjoint orbit method for real reductive groups, and generalizations of the Robinson-Schensted correspondence a la Steinberg.

This talk will present a partition-type classification, when $p > 2$, of the nilpotent K-orbits in the infinitesimal restricted symmetric spaces $\mathfrak{p}$ for groups G of classical type, and will discuss some related issues. The results in these cases are of further interest for several reasons, including the following. The classification involves some neat, but perhaps not generally well-known, matrix results. The determination of the K-orbits on $\mathcal{N}(\mathfrak{p})$ (and some results about their closures) leads to a very concrete description of the restricted nullcone $\mathcal{N}_1(\mathfrak{p}) = \mathcal{N}_1(\mathfrak{g}) \cap \mathfrak{p}$; one application of this is to simplify the difficult problem of determining the support variety of a G_1-module V, when V is also a finite-dimensional modular Harish-Chandra module. Also, knowledge of $\mathcal{N}_1(\mathfrak{p})$ may make it possible to determine a cohomological interpretation of this restricted nullcone in terms of modules for the Lie triple system $\mathfrak{p}$.

Representations of reduced enveloping algebras and cells in the affine Weyl group

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Let G be a semisimple algebraic group over an algebraically closed field of characteristic $p > 0$, and let $\mathfrak{g}$ be its Lie algebra. The crucial Lie algebra representations to understand are those associated with the reduced enveloping algebra $U_\chi(\mathfrak{g})$ for a "nilpotent" $\chi \in \mathfrak{g}^*$. We conjecture that there is a natural assignment of simple modules in a typical block to left cells in the affine Weyl group W_p (for the dual root system) which lie in the two-sided cell corresponding to the orbit of χ in Lusztig's bijection. This should respect the actions of the component group of $C_G(\chi)$ and fit naturally into Lusztig's enriched bijection which also involves characters of $C_G(\chi)$. Some evidence will be described in special cases.

Crystals and Tensor Products in Category $\mathcal{O}$

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The theory of crystal bases has proved to be a powerful tool in the study of integrable representations of quantized Kac-Moody Lie algebras. In particular, crystal bases can be used to compute the tensor product decomposition into irreducibles. In this work we approach the problem of describing the tensor product decomposition of representations belonging to BGG category $\mathcal{O}$ into indecomposables (since the category is not semisimple). We give a definition of crystals adapted for studying the indecomposables of $\mathcal{O}$. We then define a tensor product rule for these crystals and prove it describes the decomposition of the tensor product of the corresponding representations in the case of $U_q(sl_2)$.

Localization of D-modules in positive characteristic

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Let X be a smooth variety over an algebraically closed field $\mathbf{k}$ and $\mathcal{D}iff$ the sheaf of $\mathbf{k}$-algebras of differential operators on X. We say that the localization theorem for $\mathcal{D}iff$-modules holds iff there is an equivalence between the category of coherent $\mathcal{D}iff$-modules and that of $\Gamma(X, \mathcal{D}iff)$-modules of finite type.

A celebrated theorem of Beilinson and Bernstein asserts that in characteristic 0 the localization theorem for $\mathcal{D}iff$-modules holds on the flag variety. In positive characteristic, however, Kashiwara and Lauritzen have found its failure in SL_5.

In positive characteristic $\mathcal{D}iff$ admits a filtration by $\bar{\mathcal{D}}^{(m)} = \mathcal{M}od_{\mathcal{O}_{X^{(m+1)}}}(\mathcal{O}_X, \mathcal{O}_X)$, $m \in \mathbb{N}$, where $X^{(m+1)}$ is the $(m+1)$-st Frobenius twist of X. Each $\bar{\mathcal{D}}^{(m)}$ admits a lift $\mathcal{D}^{(m)}$, Berthelot's ring of arithmetic differential operators of level m. Then the localization theorem on the flag variety survives as a derived equivalence: Bezrukavnikov, Mirkovic and Rumynin have recently established an equivalence of the derived category of coherent $\mathcal{D}^{(0)}$-modules and the derived category of

$\Gamma(X, \mathcal{D}^{(0)})$-modules of finite type if the characteric is larger than the relevant Coxeter number.

We will show that the derived localization theorem holds for $\bar{\mathcal{D}}^{(m)}$-modules on the projective space $\mathbb{P}^n_{\mathbf{k}}$ iff $n < p^{m+1}$. This is a joint work with Hashimoto Yoshitake and Dmitriy Rumynin.

Mirkovic-Vilonen cycles and polytopes

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Mirkovic-Vilonen showed that certain subvarieties of the Affine Grassmanian, called Mirkovic-Vilonen cycles, index bases for representations of complex semisimple groups. Anderson observed that to each MV cycle, it is possible to associate its moment map image, called a Mirkovic-Vilonen polytope. He showed that these polytopes can be used to count tensor product multiplicities. Later, Anderson-Kogan gave a description of the MV cycles and polytopes in type A.

Here, we give a uniform description of the MV cycles and polytopes for all complex semisimple groups. Our description is in terms of the combinatorics developed by Berenstein-Zelevinsky in their tensor product multiplicities paper. However, our work does not rely on their results and so it gives a new proof of their tensor product multiplicity formula. Using our results, we are able to prove a conjecture of Anderson-Mirkovic describing how the MV polytopes change under the Braverman-Gaitsgory crystal structure.

Crystal bases for quantum affine algebras and combinatorics of young walls

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In this talk, we will introduce combinatorics of Young walls and give a realization of crystal bases for integrable representations of quantum affine algebras in terms of reduced proper Young walls.

On the Structure of Finite W-Algebras of Type A

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To every nilpotent orbit in a complex reductive finite dimensional Lie algebra $\mathfrak{g}$, Premet associates an algebra W which can be considered as a filtered deformation of the algebra of functions on the corresponding Slodowy slice. In fact, W is defined as an endomorphism algebra of the corresponding generalized Gelfand-Graev module. In this talk we will discuss the structure of W for the case $\mathfrak{g} = \mathfrak{gl}(N)$.

Exceptional groups, Jordan algebras, and higher composition laws

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We consider representations associated to maximal parabolic subgroups of exceptional algebraic groups. There is a construction due to H. Freudenthal, that allows one to construct the same pairs (group, module) starting with a vector space with an admissible cubic form.

We use the Freudenthal construction to study integer orbits of these representations, and show how they are related to higher composition laws in number theory. (Higher composition laws are recent generalizations of Gauss's law of composition of binary quadratic forms).

Crystal Structures Arising from Representations of $GL(m|n)$

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We will present results in the modular representation theory of the supergroup $GL(m|n)$. Working over a field of arbitrary characteristic, we show that the explicit combinatorics of certain crystal graphs describe aspects of an analogue of the BGG category $\mathcal{O}$. Furthermore, our approach accounts for the fact that $GL(m|n)$ has non-conjugagate Borel subgroups. As a result, Serganova's odd reflections give rise to canonical crystal isomorphisms.

Affine quivers of type $\tilde{\mathbf{A}}_n$ and canonical bases

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Let Q be an affine quiver of type $\tilde{\mathbf{A}}_n$ and $C = (a_{ij})$ be the symmetric generalized Cartan matrix associated to Q. Let U^- be the negative part of the quantized enveloping algebra attached to C. In terms of perverse sheaves on the moduli space of representations of a quiver,Lusztig constructed $U^{\leq 0} = U^0 \otimes U^-$ geometrically and gave a canonical basis $\mathbf{B}$ for U^- at the same time. The simple perverse sheaves which gives the canonical basis $\mathbf{B}$ are defined abstractly in general. For two special orientations, cyclic and the orientation defined in terms of McKay correspondence, Lusztig gave a concrete description of those simple perverse sheaves. In this talk I will describe the canonical bases in concrete terms of the simple perverse sheaves on certain algebraic varieties arising from the representation category of the quiver for arbitrary orientations.

Optimal $SL(2)$-homomorphisms

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Let G be a semisimple group over an algebraically closed field of very good characteristic for G. In the context of geometric invariant theory, G. Kempf has associated optimal cocharacters of G to an unstable vector in a linear G-representation. If the nilpotent element X of Lie(G) lies in the image of the differential of a homomorphism $SL(2) \to G$, we say that homomorphism is optimal for X, or simply optimal, provided that its restriction to a suitable torus of $SL(2)$ is optimal for X in Kempf's sense.

The talk will discuss properties of such optimal homomorphisms. For example: any two $SL(2)$-homomorphisms which are optimal for X are conjugate under the connected centralizer of X; if G is defined over the (arbitrary) subfield K of k, and if the K-rational nilpotent element X of Lie$(G)(K)$ has pth power 0, there is an optimal homomorphism for X which is defined over K.

Beilinson-Bernstein localization for quantum groups at roots of unity

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Modular representation theories afford a localization to flag varieties. Our previous work concerned Lie algebras in positive characteristic. This continuation deals with representations of quantum groups at roots of unity. The results are largely the same.

Affine Lie algebra representations and multisum identities of Rogers-Ramanujan type

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We consider the affine Lie algebra of $sl(n)$ type. It is known that the level k irreducible representation $L(k\Lambda_0)$ for this affine Lie algebra form a vetex operator algebra. Using the representation theory of this vertex operator algebra we derive some recurrence relations for the characters of level k representations. When $k = 1$, solving these recurrence relations and then taking its principal specialization we obtain a new family of multisum identities of Rogers-Ramanujan type.

Instanton counting

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Nekrasov's deformed partition functions of N=2 SUSY Yang-Mills theories are given by the integrations in the equivariant cohomology/Grothendieck groups of instanton moduli spaces over $\mathbf{R}^4$, which are quiver varieties associated with the Jordan quiver. They are analog of Donaldson invariants, and equal to Gromov-Witten invariants of certain noncompact Calabi-Yau 3-folds. We review recent results on them.

Geometric Crystals and Crystal Bases

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First, we introduce the theory of geometric crystals and unipotent crystals in Kac-Moody setting, which is a kind of geometric lifting of Kashiwara's crystal base theory. We also review tropicalization(Trop)/ultra-discretization(UD) operations. Next, we define geometric and unipotent crystal structures on Schubert cells/varieties associated with Kac-Moody groups. As applications, we give some relations with limits of affine perfect crystals and derive the braid-type isomorphisms in crystal theory by Trop/UD operations. Finally, we give a geometric crystal structure on unipotent groups and for A_n-case we describe the relation with the generalized Young tableaux.

Higher extensions for $\mathrm{SL}_2(k)$.

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In this talk we review a variant of the Lyndon-Hochschild-Serre spectral sequence for linear algebraic groups. We use this spectral sequence to show that the E_2 page for the higher extension group $\mathrm{Ext}^q_{\mathrm{SL}_2}(\Delta(\lambda), \Delta(\mu))$ is the same as the E_∞ page. In particular we get a nice recursion formula for this group and so it may be completely calculated by this method. Here $\Delta(\lambda)$ is the Weyl module for $\mathrm{SL}_2(k)$ of highest weight λ and k is an algebraically closed field of characteristic at least three. We get similar results for $\mathrm{Ext}^q_{\mathrm{SL}_2}(L(\lambda), \Delta(\mu))$ and $\mathrm{Ext}^q_{\mathrm{SL}_2}(\Delta(\lambda), L(\mu))$, where $L(\lambda)$ is the simple SL_2-module of highest weight λ.

On 2-modular representations of the symmetric groups

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Let $G = GL_n$ denote the general linear group of invertible $n \times n$ matrices with entries in an algebraically closed field F of characteristic 2. We prove the existence of certain composition factors in the restriction to $GL_{n-k} \times GL_k$ of some polynomial G-modules. As a consequence, we resolve the final case of the problem, addressed by Jantzen and Seitz, of when an irreducible modular representation of the symmetric group S_n remains irreducible upon restriction to the Young subgroup $S_{n-k} \times S_k$.

Extensions for finite groups of Lie type and the truncated induction functor

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Let G be a simple, simply connected algebraic group over a field k of positive characteristic p. Let $G(F_q)$ be the finite Chevalley group consisting of the F_q-rational points of G where $q = p^r$. In this talk we are interested in modules that are obtained by first inducing the trivial module from the finite group $G(F_q)$ to the algebraic group G and then restricting to submodules whose weights are bounded above. For sufficiently large primes these modules turn out to be semisimple. They can be used to obtain very nice and explicit formulas for the Ext-groups of the finite groups in terms of G-extensions. Here we will discuss properties of these modules for smaller primes, where they fail to be semisimple.

This is joined work with Chris Bendel and Dan Nakano.

Minimal nilpotent representations, quantizations of Slodowy slices, and the Joseph ideal

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Let e be a long root vector in a reductive Lie algebra. In my talk the algebra H_e and its finite dimensional modular counterpart will be presented by generators and relations. A link with the Joseph ideal will be established and the modular representations with p-character χ_e of dimension p^n will de described, where n=(dim G.e)/2.

Towards a Classification of Modular Tensor Categories

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The main examples of modular tensor categories (MTCs) are quotients by negligible morphisms in Andersen's categories of tilting modules over quantum groups at roots of unity. I will discuss current joint work with Z. Wang and R. Stong in which we consider the problem of classifying all MTCs with a fixed number of isomorphisms classes of simple objects (the rank). In particular we are working towards proving the conjecture that there are finitely many MTCs of a given rank n (up to equivalence). We show that the conjecture is true for ranks less than or equal to 4 and give a complete list, with the cases $n = 3$ and 4 new results. Another goal of the classification program is to understand the known constructions of finite braided tensor categories. If time permits I will also describe some new equivalences and resolve some unitarizibility questions.

Quantization of necklace Lie algebras

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Ginzburg and independently Bocklandt and Le Bruyn defined an infinite-dimensional "necklace" Lie algebra canonically associated to any quiver. Following suggestions of V. Turaev, P. Etingof, and Ginzburg, we define a cobracket, verify it gives a Lie bialgebra structure, and present an explicit Hopf algebra quantizing the necklace Lie algebra (which satisfies PBW).

The Beilinson-correspondence for quantized enveloping algebras

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Theory of the quantized flag manifold as a quasi-scheme (non-commutative scheme) has been developed by Lunts-Rosenberg. They have formulated an analogue of the Beilinson-Bernstein correspondence using their q-differential operators. In this talk we shall give its modified version using a class of q-differential operators, which is (possibly) smaller than the one by Lunts-Rosenberg.

Representations of double affine Hecke algebras

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We will review some recent progress on representations of the double affine Hecke algebras introduced by I. Cherednik.

Vanishing integrals of Macdonald polynomials
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If one integrates a Schur function s_λ over the orthogonal group, the integral is zero unless λ has all parts even. A similar statement is true for Macdonald polynomials, where one modifies the density appropriately. This modification is dictated by the representation theory of the affine Hecke algebra. This is joint work with E. Rains.

A super duality and Kazhdan-Lusztig polynomials
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We establish a direct and conceptual connection between the representation theories of Lie algebras and superalgebras (of type A), via the canonical and dual canonical bases on Fock spaces which is in turn a reformulation of Kazhdan-Lusztig theory. As a consequence, the usual parabolic Kazhdan-Lusztig polynomial of type A computes the characters of finite-dimensional irreducible modules of the general linear Lie superalgebra.

Representations of tame quivers and affine canonical bases
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The integral PBW-bases of type $A_1^{(1)}$ have been provided by P.Zhang and X.Chen according to the Auslander-Reitne quiver of the Kronecker quiver. We will arrange these bases in a geometrical order by following the method of Lusztig in the case of finite. This leads to an algebraic realization of the canonical bases of $U_q(\hat{\mathfrak{sl}_2})$. By an algebraic construction of the integral bases for a tube and an embedding of the module category of the Kronecker quiver into the module of tame quivers, we list the root vectors according to the preprojective, regular and preinjective components of the Auslander-Reiten quivers of tame quivers, and then we obtain the integral PBW-bases of the generic composition algebras. The geometric order given by the dimensions of the orbit varieties can be applied to this basis too. Then we show that the transition matrix between the PBW-basis and a monomial basis is triangular with diagonal entries equal to 1 with respect to this geometric order. Therefore we give an algebraic way, according to the idea of Lusztig for finite type, to realize the canonical bases of the quantized enveloping algebras of all symmetric affine Kac-Moody algebras.