Manhattan Mathematical Olympiad 2000
Grades 5-6

Put your name on all papers you use and turn them all in.

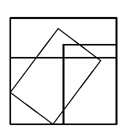
Try to solve as many problems as you can, in any order you choose. For any problem you try, give as complete an answer as you can. Include a clearly written explanation of how you found your answer and why it is true. You may use drawings or calculations as part of your justification.

Problem 1. Jane and John wish to buy a candy. However Jane needs seven more cents to buy the candy, while John needs one more cent. They decide to buy only one candy together, but discover that they do not have enough money. How much does the candy cost?

Problem 2. Farmer Jim has an 8 gallon bucket full with water. He has three empty buckets: 3 gallons, 5 gallons and 8 gallons. How can he get two volumes of water, 4 gallons each, using only the four buckets?

Problem 3. A pizza is divided into six slices. Each slice contains one olive. One plays the following game. At each move it is allowed to move an olive on a neighboring slice. Is it possible to bring all the olives on one slice by exactly 20 moves?

Problem 4. Three rectangles, each of area 6 square inches, are placed inside a 4 in. by 4 in. square. Prove that, no matter how the three rectangles are shaped and arranged (for example, like in the picture below), one can find two of them which have a common area of at least 2/3 square inches.



Manhattan Mathematical Olympiad 2000
Grades 7-8

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Problem 1. There are 6 people at a party. Prove that one can **either** find a group of 3 people in which each person is friend with the other two, **or** one can find a group of 3 people in which no two people are friends.

Problem 2. Prove that all solutions of the equation $0.001x^3 + x^2 - 1 = 0$ are irrational numbers. (A number x is said to be *irrational*, if one cannot write $x = m/n$, with m and n integer numbers.)

Problem 3. Find all 10-digit whole numbers N, such that first 10 digits of N^2 coincide with the digits of N (in the same order).

Problem 4. Is it possible to place a number of circles inside a square with side 1 cm., such that the sum of radii of all the circles is greater than 2000 cm., and no two circles have overlapping interiors?

Manhattan Mathematical Olympiad 2000
Grades 9-12

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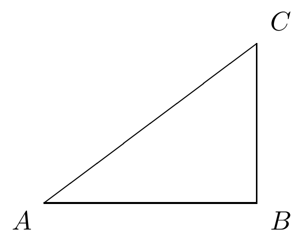
Problem 1. Prove there exists no polynomial $f(x)$, with integer coefficients, such that $f(7) = 11$ and $f(11) = 13$.

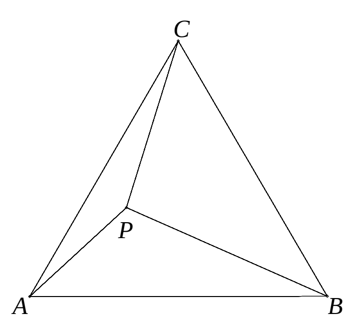
Problem 2. How many zeros are there at the end the number $9^{999} + 1$?

Problem 3. Suppose one has an unlimited supply of identical tiles in the shape of a right triangle



such that, if we measure the sides AB and AC (in inches) their lengths are integers. Prove that one can pave a square completely (without overlaps) with a number of these tiles, exactly when BC has integer length.

Problem 4. An equilateral triangle ABC is given, together with a point P inside it.



Given that $PA = 3$ cm, $PB = 5$ cm, and $PC = 4$ cm, find the side of the equilateral triangle.