Manhattan Mathematical Olympiad 2003
Grades 5-6

Write each solution on a separate piece of paper. Write your name, address and the name of your school and school teacher at the top of each paper you turn in. Explain your solution (even if you can only explain part of it, or have only part of a solution). Answers without explanations will receive no credit.

Problem 1. Cut the triangle shown in the picture into three pieces and rearrange them into a rectangle.

Problem 2. Prove that no matter what digits are placed in the four empty boxes, the eight-digit number

9999□□□□

is not a perfect square. (A *perfect square* is a whole number times itself. For example, 25 is a perfect square because $25 = 5 \times 5$.)

Problem 3. Two players play the following game, using a round table 4 feet in diameter, and a large pile of quarters. Each player can put in his turn one quarter on the table, but the one who cannot put a quarter (because there is no free space on the table) loses the game. Is there a winning strategy for the first or for the second player?

Problem 4. Form an eight-digit number, using only the digits $1, 2, 3, 4$, each twice, so that: there is one digit between the 1's, there are two digits between the 2's, there are three digits between the 3's, and there are four digits between the 4's.

Manhattan Mathematical Olympiad 2003
Grades 7-8

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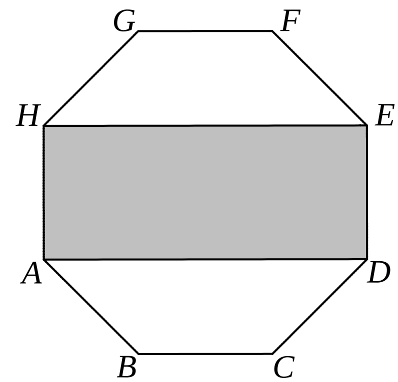


Problem 1. The polygon $ABCDEFGH$ (shown on the right) is a regular octagon. Prove that the area of the rectangle $ADEH$ is one half the area of the whole polygon $ABCDEFGH$.

Problem 2. Prove that the number

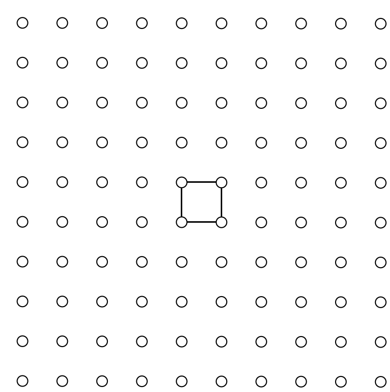
$$\frac{m}{3}+\frac{m^2}{2}+\frac{m^3}{6}$$

is integer for all integer values of m.

Problem 3. One hundred pins are arranged to form a square grid as shown. Jimmy wants to mark these pins using four letters a, b, c, d, so that:

(i) every horizontal line and every vertical line contains all four letters;

(ii) each small square (such as the one shown) has its vertices marked by four different letters.



Can he do this?

Problem 4. Prove that from any set of one hundred different whole numbers one can choose either one number which is divisble by 100, or several numbers whose sum is divisible by 100.

Manhattan Mathematical Olympiad 2003
Grades 9-12

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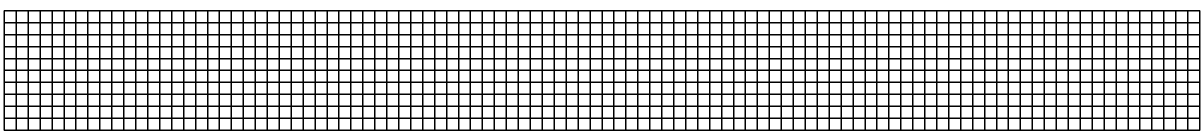
Problem 1. There are 2003 points chosen randomly in the plane in such a way that no three of them lie on a straight line. Prove that there exists a circle which contains at least three of the given points on its circumference, and no other given points inside.

Problem 2. A tennis net is made of strings tied up together which make a grid consisting of small congruent squares as shown below.



The size of the net is 100×10 small squares. What is the maximal number of edges of small squares which can be cut without breaking the net into two pieces? (If an edge is cut, the cut is made in the middle, not at the ends.)

Problem 3. Assume a, b, c are positive numbers, such that

$$a(1-b) = b(1-c) = c(1-a) = \frac{1}{4}.$$

Prove that $a = b = c$.

Problem 4. Let p and a be positive integer numbers having no common divisors except of 1. Prove that p is prime if and only if all the coefficients of the polynomial

$$F(x) = (x-a)^p - (x^p - a)$$

are divisible by p.