

1. Write your solutions on separate pieces of paper.
2. Write your name, your address, name of your school, and the school teacher at the top of each piece of paper you turn in.
3. When solving a problem explain your solution (even if you can only explain part of it, or have only part of a solution). Answers without explanations will receive no credit.

MANHATTAN MATHEMATICAL OLYMPIAD 2007

Grades 9-12

1. One throws randomly 120 squares of the size 1×1 in a 20×25 rectangle. Prove that one can still place in the rectangle a circle of the diameter equal to 1 in such a way that it does not have common points with any of the squares.
2. How many digits has the number 2^{70} (product of 70 factors of 2)?
3. Prove that the equation $x^2 - 2y^2 = 1$ has infinitely many integer solutions (i.e. there are infinitely many integers x and y which satisfy this equation).
4. Find all x, y, z which satisfy the following system of equations:

$$\begin{aligned}\frac{xy}{x+y} &= \frac{8}{3} \\ \frac{zy}{z+y} &= \frac{12}{5} \\ \frac{zx}{z+x} &= \frac{24}{7}.\end{aligned}$$