1. Write your solutions on separate pieces of paper.

2. Write your name, your address, name of your school, and the school teacher at the top of each piece of paper you turn in.

3. When solving a problem explain your solution (even if you can only explain part of it, or have only part of a solution). Answers without explanations will receive no credit.

MANHATTAN MATHEMATICAL OLYMPIAD 2006

Grades 9-12

1. Let $p_1, p_2, \ldots, p_{2006}$ be 2006 different whole numbers, all greater than 1. Prove that

$$\left(1 - \frac{1}{p_1^2}\right) \cdot \left(1 - \frac{1}{p_2^2}\right) \cdots \left(1 - \frac{1}{p_{2006}^2}\right) > \frac{1}{2}.$$

2. The billiard has the form of an $m \times n$ rectanglular table, where m and n are whole numbers. One plays a ball from the center of the table, which hits a wall of the billiard in such a way that the angle between its trajectory and the wall is 30 degrees. Prove that the ball will never hit the corner of the billiard. (Comments: You may assume that the ball is a point. The ball reflects from a wall at the same angle it hits the wall).

3. Let $a_1, a_2, a_3, \ldots$ be a sequence of whole numbers (maybe with repetitions) such that for each $k \geq 1$ there are exactly k terms of the sequence which divide k. Prove that there exists a term a_k in this sequence, which is equal to 1024.

4. Prove that there are no positive integer numbers x, y, z, k such that

$$x^k + y^k = z^k,$$

and $x < k, y < k$.