

1. Write your solutions on separate pieces of paper.
2. Write your name, your address, name of your school, and the school teacher at the top of each piece of paper you turn in.
3. When solving a problem explain your solution (even if you can only explain part of it, or have only part of a solution). Answers without explanations will receive no credit.

MANHATTAN MATHEMATICAL OLYMPIAD 2006

Grades 7-8

1. You have 10 bags of 2006 coins each and a scale which can show exact weight. You know that nine bags contain honest coins which weigh 10 grams each. The remaining bag contains false coins which weigh 11 grams each. All bags look alike, so you don't know which bag contains false coins. How to determine the bag with false coins by one weighing? (This means that you can put some coins on the scale, read the result and make a decision based on the result).
2. Is it true that if the lengths of all three altitudes of a triangle are less than 1 centimeter, then its area is less than 100 square centimeters?
3. Find the smallest possible whole number with the following property: its half is a square of some whole number, its third is a cube of some whole number and its fifth is the fifth power of some whole number.
4. Martian bus tickets have six-digit numbers, so that all tickets are numbered from 000001 to 999999. Martians think that the ticket is lucky if the sum of the first three digits is equal to the sum of the last three digits. Prove that the total sum of all 6-digit numbers which appear on the lucky tickets is divisible by 13.