

1. Write your solutions on separate pieces of paper.
2. Write your name, your address, name of your school, and the school teacher at the top of each piece of paper you turn in.
3. When solving a problem explain your solution (even if you can only explain part of it, or have only part of a solution). Answers without explanations will receive no credit.

## MANHATTAN MATHEMATICAL OLYMPIAD 2005

### Grades 5-6

1. Is there a whole number which becomes exactly 57 times less than itself when one crosses out its first digit?
2. What is the largest number of Sundays can be in one year? Explain your answer.
3. An alien from the planet Math came to Earth on Monday and said:  $A$ . On Tuesday he said  $AY$ , on Wednesday  $AYYA$ , on Thursday  $AYYAYAAAY$ . What will he say on Saturday?
4. The parliament of the country Ar consists of two houses, upper and lower, both have the same number of people. The law says that each member must vote "Yes" or "No". One day, when all members of both houses were present and voted on an important issue, the speaker informed the press that the number of members voted "Yes" was greater by 23 than the number of members voted "No". Prove that he made a mistake.

### Grades 7-8

1. Prove that having 100 whole numbers one can choose 15 of them so that the difference of any two is divisible by 7.
2. Is it possible to make a  $100 \times 100$  table of numbers such that the sum of numbers in each column is positive while the sum of numbers in each row is negative?
3. Are there six different positive odd numbers  $a, b, c, d, e, f$  such that

$$1/a + 1/b + 1/c + 1/d + 1/e + 1/f = 1?$$

4. Here is a problem given at the mathematical test at some school:

*The hypotenuse of the right triangle is 12 inches. The height (distance from the opposite vertex to the hypotenuse) is 7 inches. Find the area of the triangle.*

Everybody in the class got the answer 42 square inches, except for the two best students. Can you explain why the two best students could not get the same answer as the majority?

### **Grades 9-12**

1. In how many regions can four straight lines divide the plane? List all possible cases.

2. How many ten-digit whole numbers satisfy the following property: they have only 2 and 5 as digits, and there are no consecutive 2's in the number (i.e. any two 2's are separated by at least one 5)?

3. Prove that for any three pairwise different integer numbers  $x, y, z$  the expression  $(x - y)^5 + (y - z)^5 + (z - x)^5$  is divisible by  $5(x - y)(y - z)(z - x)$ .

4. Circle of radius  $r$  is inscribed in a triangle. Tangent lines parallel to the sides of triangle cut three small triangles. Let  $r_1, r_2, r_3$  be radii of circles inscribed in these triangles. Prove that

$$r_1 + r_2 + r_3 = r.$$