1. Write each of your solutions on a separate piece of paper.

2. Write your name, address and the name of your school and school teacher at the top of each piece of paper you turn in.

3. Explain your solution (even if you can only explain part of it, or have only part of a solution). Answers without explanations will receive no credit.

MANHATTAN MATHEMATICAL OLYMPIAD 2002

Grades 7-8

1. Prove that if an integer n is of the form $4m+3$, where m is another integer, then n is not a sum of two perfect squares (a perfect square is an integer which is the square of some integer).

2. Let us consider the sequence $1, 2, 3, ..., 2002$. Somebody choses 1001 numbers from the sequence. Prove that there are two of the chosen numbers which are relatively prime (i.e. do not have any common divisors except 1).

3. The product $1 \cdot 2 \cdot ... \cdot n$ is denoted by $n!$ and called *n-factorial.* Prove that the product

$$1!2!3!...49!51!...100!$$

(the factor 50! is missing)
is the square of an integer number.

4. Find six points $A_1, A_2, ..., A_6$ in the plane, such that for each point $A_i, i = 1, 2, ..., 6$ there are exactly three of the remaining five points exactly 1 cm from A_i.