

1. Write your solution on a separate piece of paper.
2. Write your name, address and the name of the school.
3. Explain your solution (even a part of it). Answers without explanations will receive no credit.

MANHATTAN MATHEMATICAL OLYMPIAD 2001

Grades 9-12

1. Find all integer solutions to the equation

$$x^2 + y^2 + z^2 = 2xyz.$$

2. Prove that circles which have sides of a convex quadrilateral as diameters cover its interior.

(Convex polygon is the one which contains with any two points the whole segment, joining them).

3. Let x_1 and x_2 be roots of the equation $x^2 - 6x + 1 = 0$. Prove that for any integer $n \geq 1$ the number $x_1^n + x_2^n$ is integer and is not divisible by 5.

4. How many digits has the number 2^{100} ?

5. Factorize the expression $a^3 + b^3 + c^3 - 3abc$.

6. There are n coins of the radius r on a table which is a circle of the radius R . It is known that:

- a) any two coins either touch each other or have no common points;
- b) there is no place for one more coin on the table.

Prove that

$$\frac{1}{2}\left(\frac{R}{r} - 1\right) < \sqrt{n} < \frac{R}{r}.$$