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INVERSE ACOUSTIC SCATTERING BY A LAYERED OBSTACLE

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Abstract

A uniqueness theorem is proved for the inverse acoustic scattering problem for a piecewise-homogeneous obstacle under the assumption that the scattering amplitude is known for all directions of the incident and the scattered field at a fixed frequency.

1 Introduction

The inverse problem of the scattering of a plane acoustic wave by a multilayered scatterer is studied in this work. Such a scatterer consists of a finite number of homogeneous layers, and is described in detail in Section 2. The corresponding direct scattering problem has been studied by Athanasiadis and Stratis, who have shown existence and uniqueness of its solution in [2], and have implemented the low frequency theory for the determination of the solution in [1]. Alternative formulation of a similar problem for the impedance equation has been studied in [18] and references therein. In the present work uniqueness of the solution to inverse scattering problem is established provided that the scattering amplitude is known for all directions of incident and scattered fields at a fixed frequency. The proof is based on an orthogonality result and is similar to the proofs in [7]–[12]. Similar ideas have been applied in [12] to some inverse scattering problems, inverse spectral problems, inverse geophysical problems, and some other inverse problems. The actual reconstruction of the scatterer is still an open problem. There is a number of papers in which the surface of an acoustically soft obstacle is found numerically from the scattering data, but these contain parameter–fitting schemes, rather than a reconstruction procedure based on an analytical or numerical inversion method, and they give no error estimate for the solution. Stability estimates for the recovered obstacle are not given in the present work. They were given in the papers [13], [14] for the case of a non-stratified soft scatterer.

2 The Direct Problem

Let D be a bounded closed subset of $\mathbb{R}^3$ with boundary S_0. The set D is divided into annuli–like regions D_m by surfaces S_m, where S_{m-1} surrounds S_m, and $S_{m-1} \cap S_m = \emptyset$, $m = 1, 2, \ldots, M$. The S_m, $m = 0, 1, \ldots, M$ are assumed to be $C^{1,\lambda}$–surfaces, $\lambda \in (0, 1]$. Moreover we assume that the origin is contained in D_{M+1}, the "last" region. The exterior D_0 of D , as well as each of D_m, are homogeneous isotropic media of mass density ρ_m and compressibility γ_m. The wave number k_m in each region D_m is expressed in terms of the free space (D_0) wave number $k_0 > 0$, by the relation

$$k_m^2 = \frac{\gamma_0}{\gamma_m} \frac{\rho_m}{\rho_0} k_0^2. \tag{2.1}$$

We assume that a plane acoustic wave $u^{\rm inc}$ is incident upon D; suppressing the harmonic time dependence $\exp(-i\omega t)$, ω being the angular frequency, we have

$$u^{\rm inc}(x, \alpha, k_0) = e^{ik_0 x\cdot\alpha}, \tag{2.2}$$

where α is the unit vector in the direction of propagation.

The total exterior acoustic field u_0 is given by the formula:

$$u_0(x, \alpha, k_0) = u^{\rm inc}(x, \alpha, k_0) + u^{\rm sc}(x, \alpha, k_0), \tag{2.3}$$

where $u^{\rm sc}$ is the scattered field given by

$$u^{\rm sc}(x, \alpha, k_0) = A(\alpha', \alpha, k_0) \frac{e^{ik_0 r}}{r} + o\left(\frac{1}{r}\right), \ r \to \infty \tag{2.4}$$

with $r =\mid x \mid$ and $\alpha' = \frac{x}{r}$. $A(\alpha', \alpha, k_0)$ is the scattering amplitude [6]. Formula (2.4) implies the radiation condition for the scattered field.

Let u_m be the acoustic field in D_m. Then the problem of scattering of u^{inc} by the multi–layered scatterer D, is described by the following *transmission problem*:
Find $u_m \in C^2(D_m) \cap C(\overline{D_m})$, such that

$$\Delta u_m + k_m^2 u_m = 0, \qquad \text{in} \quad D_m,\ m = 0, 1, \ldots, M+1 \tag{2.5}$$

$$\left.\begin{aligned} u_{m+1} - u_m &= 0 \\ \frac{1}{\rho_{m+1}} \frac{\partial u_{m+1}}{\partial \nu} - \frac{1}{\rho_m} \frac{\partial u_m}{\partial \nu} &= 0 \end{aligned}\right\}, \quad \text{on} \quad S_m,\ m = 0, 1, \ldots, M \tag{2.6}$$

along with (2.3) and (2.4).

It has been proved in [2] that this transmission problem has a unique solution.

Remark 2.1 The fundamental solution

$$\Phi(x, y) = \frac{e^{ik|x-y|}}{4\pi \mid x - y \mid} \tag{2.7}$$

of the Helmholtz equation $\Delta u + k^2 u = 0$ satisfies

$$\mathrm{grad}_x \Phi(x, y) = \left(ik - \frac{1}{\mid x - y \mid} \right) \Phi(x, y) \frac{x - y}{\mid x - y \mid} \tag{2.8}$$

and the asymptotic formulas

$$\Phi(x, y) = \frac{1}{4\pi \mid x - y \mid} \left[1 + O(\mid x - y \mid) \right], \quad \mid x - y \mid \to 0 \tag{2.9}$$

and

$$\mathrm{grad}_x \Phi(x, y) = \frac{x - y}{4\pi \mid x - y \mid^3} \left[1 + O(\mid x - y \mid^2) \right], \quad \mid x - y \mid \to 0. \tag{2.10}$$

If $\Phi_m(x, y)$ is the fundamental solution of $\Delta u + k_m^2 u = 0$ in D_m, $m = 0, 1, \ldots, M+1$, then $\Phi_m(x, y)$ satisfies (2.9) and (2.10). The asymptotic behavior for elliptic operators with smooth coefficients was derived by the parametrix methods in the theory of pseudodifferential operators and also by classical methods (see [5]).

We will need also the asymptotics of the principal part of the fundamental solution to elliptic equations of the second order of the divergent form (see (3.1) with $q(x) = 0$ below) for piecewise-smooth $a(x)$. We assume that $a(x)$ has a smooth surface S as a discontinuity surface, and $a(x)$ together with its derivatives is continuous up to S from inside and outside. In this case one can assume without loss of generality that S is a plane, say the plane $x_n = 0$, and the function $a(x)$ is constant a_+ if $x_n > 0$, and $a(x) = a_-$ if $x_n < 0$. The fundamental solution g is assumed continuous across S, and $a\frac{\partial g}{\partial \nu}$ is continuous across S, where ν is the normal to S, in our case ν is directed along x_n. In [17] a formula for $g(x, y)$ is obtained, where $g(x, y)$ satisfies the above transmission conditions across S and the differential equation $\text{div}[a(x)\text{grad}g] + k_0^2 q(x)g = -\delta(x - y)$ in $\mathbb{R}^n$. If $a(x)$ is piecewise-constant, then a global formula for g is given in [17], see formulas (2.11) and (2.12) below. In [4], it is claimed that g is of the form $c \mid x - y \mid^{-1}$ if $n = 3$, where $c = [(a_+ + a_-)(2\pi)]^{-1}$. This claim is not correct and we give below the correct formula. However, if y, the pole of $g(x, y)$, belongs to the interface S, then, as $|x - y| \to 0$, one obtains $g \sim c|x - y|^{-1}$, with c defined above as in [4].

Let $\gamma := (a_+ - a_-)/(a_+ + a_-)$, $\rho :=\mid \hat{x} - \hat{y} \mid$, $\hat{x} := (x_1, \dots, x_{n-1})$, $A_\pm := (4\pi a_\pm)^{-1}$, $r :=\mid x - y \mid$. We claim that the following global formula holds

$$g = A_+ \left[\frac{1}{r} + \frac{\gamma}{(\rho^2 + [\mid x_3 \mid + \mid y_3 \mid]^2)^{1/2}} \right], \ y_3 > 0, \tag{2.11}$$

$$g = A_- \left[\frac{1}{r} - \frac{\gamma}{(\rho^2 + [\mid x_3 \mid + \mid y_3 \mid]^2)^{1/2}} \right], \ y_3 < 0. \tag{2.12}$$

The idea of the proof of formulas (2.11) -(2.12) is simple: one Fourier-transforms the equation for g with respect to transversal variables $\hat{x}$ and gets an ordinary differential equation for the function $\widetilde{g}$, the Fourier transform of the above g. The solution to this equation with the transmission conditions at $x_3 = 0$ leads to the formula:

$$\widetilde{g} = \frac{\exp(-\xi \mid x_3 - y_3 \mid)}{2\xi a_+} + \gamma \frac{\exp(-\xi[\mid x_3 \mid + \mid y_3 \mid])}{2\xi a_+}, \ y_3 > 0, \tag{2.13}$$

$$\widetilde{g} = \frac{\exp(-\xi \mid x_3 - y_3 \mid)}{2\xi a_-} - \gamma \frac{\exp(-\xi[\mid x_3 \mid + \mid y_3 \mid])}{2\xi a_-}, \ y_3 < 0. \tag{2.14}$$

Inverse transforming (2.14) with respect to $\xi-$variable, one gets (2.11), (2.12). Let us note that (2.11), (2.12) hold locally for equations with variable coefficients.

Remark 2.2 The assumption that the S_m, $m = 0, 1, \dots, M$ are of class $C^{1,\lambda}$, $\lambda \in (0, 1]$, may be weakened to S_m being Lipschitz surfaces (see [15], [16]).

3 Uniqueness of Solutions to the Inverse Problem

We start this section by noticing that (2.5) may be written in the following unified way

$$(Lu)(x) := \text{div}[a(x)\text{grad}u(x)] + k_0^2 q(x)u(x) = 0, \text{ in } \bigcup_{m=0}^{M+1} D_m, \tag{3.1}$$

where

$$u_m(x) = u(x)\chi_{D_m}(x), \; m = 0, 1, \ldots, M+1 \tag{3.2}$$

$$a(x) = \begin{cases} 1 & , \; x \in D_0 \\ \dfrac{1}{\rho_m} & , \; x \in D_m \cup S_{m-1}, \; m = 1, 2, \ldots, M+1 \end{cases} \tag{3.3}$$

$$q(x) = \begin{cases} 1 & , \; x \in D_0 \\ \dfrac{1}{\rho_m}\dfrac{k_m^2}{k_0^2} & , \; x \in D_m \cup S_{m-1}, \; m = 1, 2, \ldots, M+1, \end{cases} \tag{3.4}$$

$\chi_{D_m}(x)$ being the characteristic function of D_m. Using the notation $[h(x)] = h^+(x) - h^-(x)$, $h^+(x)$ ($h^-(x)$) denoting the limit of h on S from the exterior (interior) of S, we may write (2.6) as

$$[u] = 0 \quad \text{and} \quad \left[a\frac{\partial u}{\partial \nu}\right] = 0, \text{ on } S_m. \tag{3.5}$$

Moreover, in view of (2.3) we have

$$u(x) = u^{\text{inc}}(x) + u^{\text{sc}}(x), \tag{3.6}$$

while u^{sc} is assumed to satisfy the Sommerfeld radiation condition

$$\lim_{r\to\infty} r\left(\frac{\partial u^{\text{sc}}}{\partial r} - ik_0 u^{\text{sc}}\right) = 0, \tag{3.7}$$

uniformly in all directions $\frac{x}{r}$.

Therefore, the direct transmission problem (2.3)–(2.6) may be written as the problem consisting of (3.1), (3.5), (3.6), (3.7), and will be denoted by (DTP).

Now, let

$$\mathcal{H} := H^1(D) \cap \left\{h \in H^1(\overline{D_0}_{loc}) : \; h \text{ satisfies (3.6) and (3.7)}\right\}. \tag{3.8}$$

We denote by H_0 the set of h which satisfy (3.8) with the condition that h solves (3.6) dropped and (3.7) replaced for the elements of H_0 by the condition that h vanishes near infinity. For the use of such spaces we refer to [3]. We are now in a position to give

Definition 3.1 *(DTP) has a weak solution $u \in \mathcal{H}$ iff*

$$\int_{\mathbb{R}^3} \{-a(x)\, \mathrm{grad} u(x) \cdot \mathrm{grad} v(x) + \, k_0^2 \, q(x) \, u(x) \, v(x)\} \, dx = 0, \ \forall v \in H_0. \quad (3.9)$$

Let us note that the transmission conditions (3.5) are incorporated in (3.9).

The *inverse transmission problem* consists of finding D (i.e. the surfaces S_m, $m = 0, 1, \ldots, M$), given the scattering amplitude $A(\alpha', \alpha, k_0)$ for all directions of incident and scattered fields at a fixed $k_0 > 0$. The constant parameters k_m^2 and ρ_m of the m-th layer are assumed known but the boundaries of the layers are not known when the inverse problem is studied.

In this paper we study the uniqueness of the solutions to the inverse transmission problem. The main uniqueness result is proved by an approach similar to that used in [7] – [12].

For the formulation of the results, we assume that there are two multi–layered scatterers $D^{(1)}$ and $D^{(2)}$. Each $D^{(j)}$ is stratified by surfaces $S_m^{(j)}$ into layers $D_m^{(j)}$, $m = 0, 1, \ldots, M$ and $j = 1, 2$, as described in Section 2.

We start with the following orthogonality result.

Proposition 3.1 *Let us assume that*

$$A^{(1)}(\alpha', \alpha, k_0) = A^{(2)}(\alpha', \alpha, k_0), \quad (3.10)$$

for all α', α on the unit sphere, and a fixed $k_0 > 0$. Let w_j be the solution of $L^{(j)} w_j = 0$ in Ω, $j = 1, 2$, satisfying (3.6)-(3.7), where Ω is any domain containing $\overline{D^{(1)}} \cup \overline{D^{(2)}}$. Then

$$\int_{D^{(1)}} \{-a_1(x)\, \mathrm{grad} w_1(x) \cdot \mathrm{grad} w_2(x) + k_0^2 \, q_1(x) \, w_1(x) \, w_2(x)\} \, dx =$$
$$\int_{D^{(2)}} \{-a_2(x)\, \mathrm{grad} w_1(x) \cdot \mathrm{grad} w_2(x) + k_0^2 \, q_2(x) \, w_1(x) \, w_2(x)\} \, dx \quad (3.11)$$

Proof. Let $w := w_1 - w_2$, $a := a_1 - a_2$, and $q := q_1 - q_2$. Subtract from (3.1) with $w = w_1$, $a = a_1$, $q = q_1$ this equation with $w = w_2$, $a = a_2$, $q = q_2$, to get

$$\mathrm{div}(a_1 \, \mathrm{grad} w) + k_0^2 \, q_1 \, w = -\mathrm{div}(a \, \mathrm{grad} w_2) - k_0^2 \, q \, w_2. \quad (3.12)$$

Let $\Omega_0 := \mathbb{R}^3 \setminus (\overline{D^{(1)}} \cup \overline{D^{(2)}})$. We have

$$\Delta w + k_0^2 w = 0, \ \text{in } \Omega_0. \quad (3.13)$$

From (3.10), (2.3), (2.4) we derive that

$$w(x) = O\left(\frac{1}{\mid x \mid^2}\right), \quad \mid x \mid \to \infty, \quad (3.14)$$

whereby

$$w(x) = 0 \quad \text{for} \quad |\, x \,| \quad \text{large.} \tag{3.15}$$

Hence, by the unique continuation principle, we obtain

$$w(x) = 0\,, \quad x \in \Omega_0. \tag{3.16}$$

Multiplying (3.12) by $\varphi \in H_0^1(\Omega)$ and integrating over Ω we get

$$\int_\Omega \{-a_1 \,\mathrm{grad} w \cdot \mathrm{grad}\varphi + k_0^2\, q_1\, w\, \varphi\} dx = \int_\Omega \{a \,\mathrm{grad} w_2 \cdot \mathrm{grad}\varphi - k_0^2\, q\, w_2\, \varphi\} dx \tag{3.17}$$

Now, let Ω_* be any open bounded set in $\mathbb{R}^3$, containing $D^{(1)} \cup D^{(2)}$, such that $\overline{\Omega}_* \subset \Omega$. Let $\psi \in C_0^\infty(\Omega)$, $\psi = 1$ in Ω_*. Then $\varphi := \psi\, w_1 \in H_0^1(\Omega)$. Hence

$$\begin{aligned} &\int_\Omega \{-a_1 \,\mathrm{grad} w \cdot \mathrm{grad}\varphi + k_0^2\, q_1\, w\, \varphi\} dx = \\ &\int_\Omega (L^{(1)}\varphi)\, w\, dx = \int_\Omega (L^{(1)} w_1)\, w\, dx = 0, \end{aligned} \tag{3.18}$$

and so

$$\int_\Omega \{a \,\mathrm{grad} w_2 \cdot \mathrm{grad}\varphi - k_0^2\, q\, w_2\, \varphi\} dx = 0. \tag{3.19}$$

But in Ω_0 we have $a = 0$, $q = 0$, while in $D^{(1)} \cup D^{(2)}$ we may take $\varphi = w_1$, whereby (3.11) follows. □

We now prove:

Theorem 3.1 *If (3.10) holds, then* $D^{(1)} = D^{(2)}$.

Proof. As mentioned in the description of the inverse transmission problem, it suffices to show that $S_m^{(1)}$ coincides with $S_m^{(2)}$, $m = 0, 1, \ldots, M$. Suppose, to the contrary, that there is a $m_0 \in \{0, 1, \ldots, M\}$ such that $S_{m_0}^{(1)}$ does not coincide with $S_{m_0}^{(2)}$. Let $\widetilde{D}^{(j)}$, $j = 1, 2$, denote the domain circumscribed by $S_{m_0}^{(j)}$. Assume that $\widetilde{D}^{(1)}$ is not contained in $\widetilde{D}^{(2)}$. Without loss of generality, we suppose that $m_0 = 0$ (and hence $\widetilde{D}^{(j)} = D^{(j)}$). Let z be on the part of $S_0^{(1)}$ which is not inside $D^{(2)}$. Consider a ball B_z, centered at z, having no common points with $\overline{D^{(2)}}$, while its part inside $D^{(1)}$ lies in the first layer $D_1^{(1)}$, without touching $S_1^{(1)}$. Let $w_j(x, y)$ be the Green function of $L^{(j)}$, and $y \in \Omega_0$. Since the Green function with y away from B_z can be approximated with arbitrary accuracy in $H^2(B_z)$ by scattering solutions (see [12],[9]), one can use as w_j in (3.11) the functions $w_j(x, y)$. Write (3.11) as

$$\begin{aligned} &\int_{D^{(1)} \cap B_z} \{-a_1(x) \,\mathrm{grad} w_1(x,y) \cdot \mathrm{grad} w_2(x,y) + k_0^2\, q_1(x)\, w_1(x,y)\, w_2(x,y)\}\, dx = \\ = \;& \int_{D^{(1)} \setminus B_z} \{a_1(x) \,\mathrm{grad} w_1(x,y) \cdot \mathrm{grad} w_2(x,y) - k_0^2\, q_1(x)\, w_1(x,y)\, w_2(x,y)\}\, dx + \\ + \;& \int_{D^{(2)}} \{-a_2(x) \,\mathrm{grad} w_1(x,y) \cdot \mathrm{grad} w_2(x,y) + k_0^2\, q_2(x)\, w_1(x,y)\, w_2(x,y)\}\, dx. \end{aligned} \tag{3.20}$$

Note that the right-hand side of (3.20) is bounded as $y \to z$.

When $y \to z$, we observe that $O(\mid x-y \mid)$ is small, for $x \in D^{(1)} \cap B_z$. Hence, by formulas (2.11)-(2.12) for the principal parts of the fundamental solutions of the second-order elliptic differential equations with discontinuous coefficients we obtain

$$\left| \int_{D^{(1)} \cap B_z} k_0^2 \, q_1(x) \, w_1(x,y) \, w_2(x,y) \, dx \right| \leq \, c_1, \tag{3.21}$$

and, using formulas (2.11)-(2.12) again, we get:

$$\int_{D^{(1)} \cap B_z} a_1(x) \, \mathrm{grad} w_1(x,y) \cdot \mathrm{grad} w_2(x,y) \, dx \geq \, c_2 \int_{D^{(1)} \cap B_z} \frac{dx}{\mid x-y \mid^4}, \tag{3.22}$$

where c_1 and c_2 are positive constants.

The right-hand side of (3.22) tends to $+\infty$ as $y \to z$ if $z \in \mathbb{R}^3$, and therefore all the terms in (3.20), except one, are bounded. Thus, a contradiction. If the dimension of the space is greater than 3, the conclusion of Theorem 3.1 remains valid and the argument needs only a slight modification.

Hence $D^{(1)}$ must be contained in $D^{(2)}$. Arguing similarly we obtain that $D^{(2)}$ must be contained in $D^{(1)}$, and therefore that $D^{(1)} = D^{(2)}$. Hence $S_0^{(1)}$ coincides with $S_0^{(2)}$. Now, we follow the above procedure stepwise; repeating it, we conclude that $S_m^{(1)}$ coincides with $S_m^{(2)}$ for all $m = 0, 1, \ldots, M$, q.e.d. $\square$

Remark 3.1 The previous result and proof remain valid for variable coefficients a, q belonging to the class of piecewise-smooth functions with a non–zero jump at S_m. Moreover, our argument shows that $a_1 = a_2$ at S_m even if one does not assume a priori that $a_1 = a_2$ in the layers.

If $a(x)$ in (3.1) is H^2-smooth, rather than piecewise-H^2-smooth, then it was proved in [9] (see also [12]) that the scattering data at two frequencies determine both $a(x)$ and $q(x)$ in (3.1) uniquely. Stability of the solution to the inverse obstacle problem was studied in [6], [13] and [14].

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