Name:

MATH 512 Intro to Modern Algebra – **Exam II**

Wednesday, October 26, 2005

Check that that you have all four pages - note that the pages are double-sided

---

1. (17 points) For the permutations

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 3 & 4 & 7 & 6 & 8 & 5 & 1 & 2 \end{pmatrix}, \qquad \tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 5 & 1 & 8 & 7 & 2 & 6 & 3 & 4 \end{pmatrix}.$$

(a) Find $\tau^{-1}\sigma$.

(b) Write σ as a product of disjoint cycles.

(c) Write σ as a product of transpositions. Is σ even, odd, neither or both?

(d) What is the order of $< \sigma >$?

(e) How many cosets does $H =< \sigma >$ have in S_8.

2. (7 points) Suppose that $G = G_1 \times G_2$ where G_1 and G_2 are groups with $|G_1| = 15$ and $|G_2| = 10$.
(i) What is $|G|$?

(ii) Show that G can not be cyclic.

PG SCORE/24

3. (22 points) Circle True (T) or False (F).

T F (a) $aH = bH$ iff $Ha = Hb$.

T F (b) A cycle of odd length is an even permutation.

T F (c) S_6 has no subgroup of order 7.

T F (d) $\mathbb{Z}_{75} \simeq \mathbb{Z}_3 \times \mathbb{Z}_5 \times \mathbb{Z}_5$.

T F (e) Apart from the identity, the elements of $\mathbb{Z} \times \mathbb{Z}_{15}$ all have infinite order.

T F (f) If $K < H < G$ with $(G : H) = 5$ and $(H : K) = 10$ then $|G| = 50$.

T F (g) S_8 has a cyclic subgroup of order 12.

T F (h) If H, K are subgroups of G then $H \cup K \leq G$.

T F (i) The smallest subgroup containing a and b is $\{a^i b^j \ : \ i, j \in \mathbb{Z}\}$.

T F (j) The cosets of $< 4 >$ in $\mathbb{Z}_{12}$ are $\{0, 1, 2, 3\}, \{4, 5, 6, 7\}, \{8, 9, 10, 11\}$.

T F (k) Every group has a cyclic subgroup.

4. (12 points) Suppose that H is a subgroup of S_n containing the odd permutation σ.

(a) Define what it means for a permutation τ in S_n to be even. Show that τ^{-1} is even iff τ is even.

(b) Prove that $K = H \cap A_n$ is a subgroup of H.

(c) Show that any odd permutation ρ in H can be written in the form $\rho = \sigma\tau$ for some τ in K.

(d) What is the index $(H : K)$?

PG SCORE/34

5. (16 points) (a) What is the order of 35 in $\mathbb{Z}_{150}$?

(b) What is the subgroup of $\mathbb{Z}$ generated by the set $\{24, 60, 90\}$?

(c) What is the subgroup of $\mathbb{Z}_{100}$ generated by the set $\{24, 60, 90\}$?

(d) What is the order of the element $(3, 5, 6)$ in the group $\mathbb{Z}_{30} \times \mathbb{Z}_7 \times \mathbb{Z}_{20}$?

6. (7 points) (a) If H is a subgroup of a finte group G then Lagrange's Theorem states that:

(b) Suppose that G is a group of prime order p. Prove that G is cyclic.

PG SCORE/23

7. (11 points) a) What are the subgroups of $\mathbb{Z}_{18}$?

(b) Which elements generate $\mathbb{Z}_{18}$?

(c) Describe the automorphisms of $\mathbb{Z}_{18}$.

8. (8 points) For the group $G = \{e, a, b, c, d, f, g, h\}$ given by the table:

$*$	e	a	b	c	d	f	g	h
e	e	a	b	c	d	f	g	h
a	a	b	c	e	g	h	f	d
b	b	c	e	a	f	d	h	g
c	c	e	a	b	h	g	d	f
d	d	h	f	g	e	b	c	a
f	f	g	d	h	b	e	a	c
g	g	d	h	f	a	c	e	b
h	h	f	g	d	c	a	b	e

(a) Find the left cosets of $H = \{e, h\}$ in G.

(b) Find the right cosets of $H = \{e, h\}$ in G.

PG SCORE/19