

Name:

MATH 510 Discrete Math – **Exam II**
Wednesday, July 21, 2004

Check that that you have all four pages. Show all your work and reasoning.

1. (18 points) Consider the nonhomogeneous recurrence relation

$$h_n = 6h_{n-1} - 9h_{n-2} + 8, \quad h_0 = 7, h_1 = 23.$$

(a) Give the general solution of the related homogeneous recurrence $h_n = 6h_{n-1} - 9h_{n-2}$.

(b) Find a particular solution to the nonhomogeneous recurrence.

(c) Solve the nonhomogeneous recurrence.

2. (8 points) Evaluate the coefficient of $x^2y^3z^6$ in the multinomial expansion of $(2x - y + z^3)^7$?

3. (10 points) Find the generating function $g(x) = \sum_{n=0}^{\infty} h_n x^n$ for the recurrence relation

$$h_n = 3h_{n-1} + 10h_{n-2}, \quad h_0 = 4, h_1 = 13.$$

4. (12 points) (a) The inclusion-exclusion principle for three sets $A, B, C \subseteq S$ states that

$$|\overline{A} \cap \overline{B} \cap \overline{C}| =$$

(b) How many anagrams of PRUDENTIAL do not contain any of the words RAT, PIN or DUEL.

5. (12 points) Find the number of integer solutions to

$$x_1 + x_2 + x_3 + x_4 = 15, \quad 0 \leq x_1, x_2, x_3, x_4 \leq 5.$$

6. (12 points) How many ways can you put 6 non-attacking rooks on the 6-by-6 chessboard with forbidden positions shown.

X					
	X				
	X				
			X		
				X	
			X	X	

7. (8 points) Let h_n denote the number of ways in which the squares of a 2-by- n chessboard can be covered by dominoes so that no two adjacent dominoes are placed vertically. Find a recurrence relation satisfied by h_n . Do not try to solve it!

8. (12 points) Let h_n denote the number of ways to choose n pieces of fruit from a choice of (identical and unlimited) apples, peaches, oranges and pineapples, where the oranges come in packs of five and you want at most 4 pineapples.

(a) Give the generating function $g(x) = \sum_{n=0}^{\infty} h_n x^n$ and simplify.

(b) Obtain a formula for h_n .

9. (8 points) Five people arrive at a party wearing similar coats. How many different ways can they be handed back so that everyone leaves with the wrong coat. You can use formulae to evaluate D_n , Q_n etc. as long as you make clear what you are doing.