Name:

MATH 510 Discrete Math – **Exam II**

Friday, November 2, 2007

Check that that you have all four pages. You do not need to evaluate factorials or binomial coefficients.

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1. (14 points) (a) Give the general solution of the homogeneous recurrence $h_n = 8h_{n-1} - 16h_{n-2}$.

(b) Find a particular solution of the nonhomogeneous recurrence $h_n = 8h_{n-1} - 16h_{n-2} + 27$.

(c) Use (a) and (b) to solve the recurrence $h_n = 8h_{n-1} - 16h_{n-2} + 27, \quad h_0 = 8,\ h_1 = 31$.

2. (6 points) Six couples attend a dance. How many ways can the men and women be paired up so that no one is dancing with their spouse? You can use any formula to evaluate D_n, Q_n etc.

3. (6 points) Give a combinatorial explanation of $\displaystyle\sum_{j=1}^{n} j\binom{n}{j} = n2^{n-1}$.

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4. (14 points) (a) Let h_n denote the number of ways to choose n pieces of fruit from a choice of apples, pears, bananas and pineapples, where the bananas are in bunches of five and you want at most four pineapples. Give the generating function $g(x) = \sum_{n=0}^{\infty} h_n x^n$. Simplify and deduce a formula for h_n.

(b) Give the generating function $g(x) = \sum_{n=0}^{\infty} h_n x^n$ for the sequence satisfying the following recurrence. Don't try to solve it.

$$h_n = 2h_{n-1} + 15h_{n-2}, \quad h_0 = 3,\ h_1 = 7.$$

5. (10 points) (a) The inclusion-exclusion principle for three sets $A, B, C \subseteq S$ states that

$|\overline{A} \cap \overline{B} \cap \overline{C}| =$

(b) Count the number of integer solutions to the equation:

$$x_1 + x_2 + x_3 + x_4 = 15,\ \ 0 \le x_1 \le 8,\ 0 \le x_2 \le 4,\ 0 \le x_3 \le 6,\ 0 \le x_4.$$

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6. (6 points) Let h_n denote the number of ways in which the squares of a one-by-n chessboard can be colored using the colors red, white, blue, purple and yellow so that no two yellow squares are adjacent. Find a recurrence relation satisfied by h_n. Don't solve it.

7. (10 points) Eight balls are arranged in a line. If two are red, two blue, two green and two yellow how many ways can they be arranged with no balls of the same color adjacent? (Hint: express this as $|\overline{A_1} \cap \overline{A_2} \cap \overline{A_3} \cap \overline{A_4}|$ for suitable sets A_1, A_2, A_3, A_4).

8. (10 points) How many ways can you put 5 non-attacking rooks on the 5-by-5 chessboard with forbidden positions shown.

X				
X	X			
X				
		X		X
		X	X	X

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9. (12 points)(a) The binomial theorem states that $(1+x)^n = \sum_{j=0}^{n}$ _____.

Evaluate each of the following sums:

(b) $\sum_{j=0}^{n} (-1)^j \binom{n}{j} 3^j$

(c) $\sum_{j=0}^{n} \binom{n}{j} \frac{(2^{j+1}-1)}{j+1}$

10. (6 points) What is the coefficient of $x^3y^3z^2w^2$ in $(2x-y-z-3w)^{10}$?

11. (8 points) (a) Pascal's formula states that

$$\binom{n}{r} = \binom{\quad}{\quad} + \binom{\quad}{\quad}.$$

(b) Use induction to prove the following identity for all integers $n \geq 5$:

$$\sum_{j=5}^{n} \binom{j}{5} = \binom{n+1}{6}.$$

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