Name:

MATH 510 Discrete Math – **Exam I**

Wednesday, September 19, 2007

Check that that you have all three pages - page two is on the back of page one.
Show all your work and reasoning. You do not need to evaluate factorials or binomial coefficients.
If you apply the box principle make sure you indicate clearly what boxes you are using.

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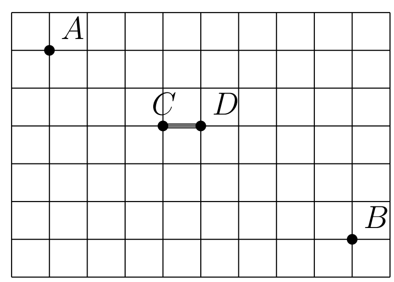
1. (15 points) How many 6 character passwords can be made using only $1, 3, 5, 7, a, b, c, d, e$ or f:

(a) Assuming no character is used more than once.

(b) Repeats are allowed as long as they are not adjacent.

(c) Repeats are allowed but the password must contain an equal number of letters and digits.

2. (10 points) The streets of a city run North-South and East-West. Assume in the following that you always take the shortest route.



(a) How many routes are there from A to B?

(b) What if the block from C to D is closed off for resurfacing?

3. (5 points) Fifteen people are lined up in three rows of five for a photo. How many ways can they be arranged if the three tallest must stand in the back row and the two shortest in the front row?

4. (5 points) Two eleven member soccer teams are to be selected from 34 students, 18 of them girls and 16 boys. How many ways to select the teams if one team is to be all girl?

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5. (5 points) Count the permutations of the letters of the word MAMMALIAN.

6. (10 points) Eight people are to be arranged around a table. How many circular arrangements if:
(a) Anne refuses to sit next to or opposite her ex Brad?

(b) They are four couples and spouses should be seated next to each other?

7. (5 points) How many ways can 4 non-attacking rooks be placed on a 6×8 chess-board?

8. (10 points) Twelve indistinguishable snickers bars are be distributed amongst 5 children.
(a) How many ways if each child must receive at least one bar?

(b) How many ways if Jake is allowed at most 6 bars? Not everyone need receive a bar.

9. (5 points) Count the number of integer solutions to

$$x_1 + x_2 + x_3 + x_4 + x_5 = 15$$

with $x_1 \geq 2$, $x_2 \geq 0$, $x_3 \geq -4$, $x_4 \geq -2$, $x_5 \geq 0$.

10. (5 points) How many numbers between 1 and 99999 contain exactly two sevens (and the other digits non-sevens eg 17376, 377, 71007)?

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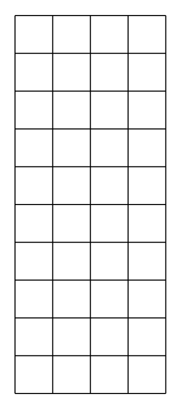
11. (10 points)(a) Anne chooses 13 numbers from 1,...,24. Prove that her selection includes two numbers with one dividing the other.

(b) Bob chooses 55 numbers from 1 to 100. Prove that he has chosen four numbers whose digits add up to the same amount.

12. (5 points) The edges of a k_5 have been colored red or blue. Suppose that at one vertex three of the edges are red and one blue. Show that there must be a red or blue k_3.

13. (5 points) How many positive whole numbers divide 77000?

14. (5 points) Prove that a 10×4 board perfectly covered by twenty dominoes must have a fault line (i.e. can be cut vertically or horizontally into two parts without cutting a domino).



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