Name:

MATH 510 Discrete Math – Exam II

Wednesday, November 1, 2006

Check that that you have all four pages. You do not need to evaluate factorials or binomial coefficients.

---

1. (18 points) Solve the recurrence relations:
(a) $h_n = h_{n-1} + 6h_{n-2}, \quad h_0 = 3, h_1 = 4.$

(b) $h_n = 10h_{n-1} - 25h_{n-2}, \quad h_0 = 2, h_1 = 15.$

2. (8 points) Give a combinatorial explanation of $\displaystyle\sum_{j=1}^{n} j\binom{2n}{j}\binom{2n}{n-j} = 2n\binom{4n-1}{n-1}.$

PG SCORE/26

3. (6 points) What is the coefficient of $x^4y^3z^2$ in the expansion of $(3x - 2y + z + 5w)^9$?

4. (10 points) By the inclusion-exclusion principle

$$|A \cup B \cup C| =$$

How many of the numbers 1,2,3,...,1000 are multiples of 4, 6 or 15?

5. (8 points) Seven people are standing in a queue. How many ways can they be rearranged so that no one is standing behind the same person. You can use formulae to evaluate D_n, Q_n etc., as long as you make clear what you are doing.

6. (10 points) (a) The inclusion-exclusion principle for three sets $A, B, C \subseteq S$ states that

$$|\overline{A} \cap \overline{B} \cap \overline{C}| =$$

(b) How many ways can you choose 14 pieces of fruit from a choice of (identical) apples, peaches, oranges and pears if there are more than 14 apples but only 6 peaches, 7 oranges and 4 pears?

PG SCORE/34

7. (10 points) A die is rolled 8 times. How many of the 6^8 different outcomes contain a 1,2,3 and 4? (i.e. at least one of each). Hint: express this as $|\overline{A_1} \cap \overline{A_2} \cap \overline{A_3} \cap \overline{A_4}|$ for suitable sets A_1, A_2, A_3, A_4.

8. (10 points) How many ways can you put 5 non-attacking rooks on the 5-by-5 chessboard with forbidden positions shown.

X	X			
	X			
X				
		X	X	
			X	X

PG SCORE/20

9. (12 points) Use the binomial theorem to evaluate each of the following:

(a) $\sum_{j=0}^{n} j\binom{n}{j}2^{j-1}$

(b) $\sum_{j=0}^{n} \binom{n}{j}\frac{2^{j+1}}{j+1}$

10. (10 points) Recall the Fibonacci sequence $f_0, f_1, f_2, f_3, f_4, f_5, f_6, f_7, f_8, ... = 0, 1, 1, 2, 3, 5, 8, 13, 21, ...$ Conjecture a formula for

$$1 + f_0 + f_2 + f_4 + \cdots + f_{2n}.$$

Give a proof by induction of your formula.

PG SCORE/22