

Name:

MATH 510 Discrete Math – Exam I

Wednesday, September 20, 2006

Check that that you have all three pages - page two is on the back of page one.
Show all your work and reasoning. You do not need to evaluate factorials or binomial coefficients.
If you apply the box principle make sure you indicate clearly what boxes you are using.

1. (15 points) Six garden gnomes in a row are each painted red, green, blue or yellow. How many ways if:
(a) No restrictions

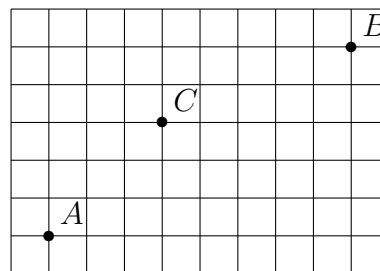
(b) No two adjacent gnomes are painted the same color.

(c) Exactly three of the gnomes are painted yellow, but no other color is repeated.

2. (10 points) The streets of a city run North-South and East-West. Assume in the following that you always take the shortest route.

(a) How many routes are there from A to B?

(b) What if intersection C is blocked by an accident?



3. (5 points) Out of 12 employees, 5 are to be assigned to sales, 4 to advertising, and 3 fired. How many ways to do this if only 6 of the employees are qualified for advertising?

4. (5 points) A group of six room-mates observe that it is impossible to find three of them who all like each other. Assuming that two people either both like or both dislike each other, what does Ramsey's Theorem tell you?

5. (5 points) Count the permutations of the letters of the word BALACLAVA.

6. (10 points) Ten diplomats are to be arranged around a table.

(a) How many circular arrangements are possible?

(b) What if the US diplomat refuses to sit next to the Cuban or the Iranian diplomat?

7. (8 points) (a) How many ways can 5 non-attacking rooks be placed on a 8×8 chess-board?

(b) What if 2 of the rooks are red and 3 blue?

8. (5 points) Ten apples and an orange are to be distributed amongst 4 children. How many ways to assign the fruit?

9. (10 points) Count the number of integer solutions to

$$x_1 + x_2 + x_3 + x_4 + x_5 = 10$$

(a) with $x_1 \geq 3$, $x_2 \geq 0$, $x_3 \geq -2$, $x_4 \geq 0$, $x_5 \geq 0$.

(b) with $0 \leq x_1 \leq 5$, $x_2, x_3, x_4, x_5 \geq 0$.

10. (7 points) How many four digit numbers with distinct digits are odd?

11. (10 points) Bob chooses 52 (distinct) numbers from $100, 101, 102, 103, 104, \dots, 199$.

(a) Prove that his choice must include two numbers whose sum is 300.

(b) Prove that he has also chosen three numbers whose digits add up to the same amount.

12. (10 points) (a) What is the magic sum of a 3×3 magic square? Show that if each entry a in a 3×3 magic square is replaced by $10 - a$ then the result is still a magic square.

(b) A student wants to arrange the numbers from 1 to 20 in a 4×5 grid so that sum of any row or column is the same. Prove that this is not possible!

13. (5 points) Prove that a 10×10 board can not be covered by twenty-five 4-ominoes.

