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MATH 510 Discrete Math – **Final Exam**

Wednesday, December 17, 2003

Check that that you have all five pages. Show all your work and reasoning.
If you have sufficient time evaluate factorials and binomial coefficients.
If you use the box principle indicate clearly what boxes you are using.

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1. (10 points) Solve the recurrence relation $h_n = 6h_{n-1} - 9h_{n-2}, \quad h_0 = 5, h_1 = 21.$

2. (8 points) Give the generating function $g(x)$ for the recurrence $h_n = 2h_{n-1} + 3h_{n-2}$, $h_0 = 1, h_1 = 4.$

3. (6 points) A company has 12 job applicants from which to select 4 programmers and 3 management trainees. How many ways can they do this if only 7 of the applicants are qualified to be programmers? Assume no one is offered more than one job and any of the applicants can be management trainees.

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4. (6 points) In the multinomial expansion of $(2x - y + 5z)^8$ what is the coefficient of x^2y^5z?

5. (6 points) Find the number of integer solutions to $x_1 + x_2 + x_3 = 20$, $x_1 \geq 3, x_2 \geq 0, x_3 \geq 0$.

6. (6 points) Eight diplomats are to be seated around a circular table (places indistinguishable). How many circular arrangements are possible if the US representative refuses to sit next to the French representative?

7. (10 points) Use the deferred acceptance algorithm to find the women-optimal stable complete marriage for the preferential ranking matrix

$$\begin{bmatrix} 2,1 & 3,4 & 1,3 & 4,1 \\ 4,2 & 2,3 & 1,4 & 3,4 \\ 3,3 & 1,1 & 2,1 & 4,2 \\ 4,4 & 1,2 & 2,2 & 3,3 \end{bmatrix}$$

Here rows A, B, C, D are the women and columns a, b, c, d are the men.

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8. (10 points) Use the inclusion-exclusion principle to count the number of permutations of the letters of word FLUSTERING that do not contain any of the words FLU, GIN, REST.

9. (10 points) (a) Is it possible to draw a general graph with degree sequence $(4, 4, 4, 4, 3)$? Explain.

(b) Does a (simple) graph G with degree sequence $(4, 4, 4, 3, 3)$ always have a Hamilton cycle? Explain.

10. (12 points) Let h_n denote the number of ways to choose n pieces of fruit from a choice of (identical and unlimited) apples, oranges, papayas, melons and pineapples, where the oranges come in packs of three, the papaya in packs of two and you want at most one melon and at most two pineapples.

(a) Give the generating function $g(x) = \sum_{n=0}^{\infty} h_n x^n$ and simplify.

(b) Obtain a formula for h_n.

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11. (10 points) How many ways can you put 6 non-attacking rooks on the 6-by-6 chessboard with forbidden positions shown.

X					
X	X				
X	X				
			X		
				X	
			X		

12. (6 points) A caterer makes and wraps 100 turkey, ham, cheese and peanut butter sandwiches but neglects to label which is which. What is the minimum number of random sandwiches you would have to take to guarantee that you have at least 6 sandwiches of the same type?

13. (6 points) Bob picks 11 distinct numbers from 1 to 20. Prove that amongst the numbers selected are two whose difference is 10.

14. (12 points) Give six non-isomorphic trees of order 6.

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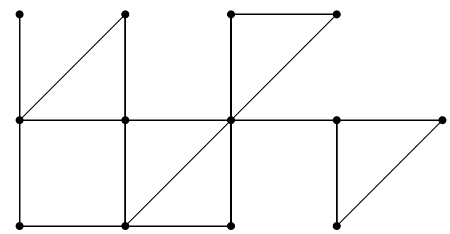
15. (12 points)

(i) Does G have a closed Euler trail? Explain

G



(ii) Does G have an open Euler trail? If so circle the start and finish of the trail.

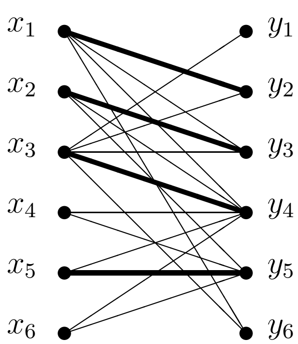
(iii) Label any bridges b or pendent edges p.

16. (10 points) Use the binomial theorem $(1+x)^n = \sum_{j=0}^{n}$ _________ to evaluate the sum $\sum_{j=0}^{n} \binom{n}{j} \frac{2^{j+1}}{j+1}$

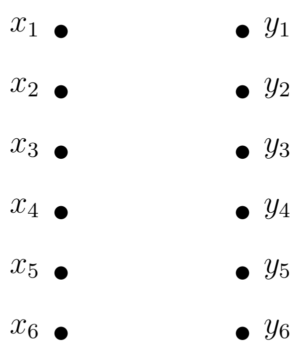
17. (10 points) For the bipartite graph and matching M:

(a) Find an M-alternating chain and hence a new matching M' with 5 edges.

M



M'



(b) Show that its a max matching by finding a cover S with 5 vertices:

$S = \{ \qquad\qquad\qquad\qquad \}$

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