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MATH 510 Discrete Math – Exam II

Wednesday, November 12, 2003

Check that that you have all four pages. Show all your work and reasoning.
If you have sufficient time evaluate factorials and binomial coefficients.

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1. (18 points) Solve the recurrence relations:
(a) $h_n = 4h_{n-1} + 5h_{n-2}, \quad h_0 = 3, h_1 = 3.$

(b) $h_n = 4h_{n-1} - 4h_{n-2}, \quad h_0 = 2, h_1 = 10.$

2. (6 points) Give a combinatorial explanation of $\binom{10}{3} = \binom{6}{3}\binom{4}{0} + \binom{6}{2}\binom{4}{1} + \binom{6}{1}\binom{4}{2} + \binom{6}{0}\binom{4}{3}$.

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3. (12 points) In the multinomial expansion of $(x - y + 2z + w)^{10}$:
(a) What is the coefficient of $x^3y^5z^2$?

(b) What is the coefficient of $x^2y^5z^2w^2$?

(c) How many terms are there in the expansion?

4. (10 points) (a) State a formula or other way to compute D_n the number of ways of deranging n things:

(b) Seven people each order a different sandwich. How many ways can the sandwiches be handed out so that no one gets the correct sandwich?

5. (10 points) (a) The inclusion-exclusion principle for three sets $A, B, C \subseteq S$ states that

$|\overline{A} \cap \overline{B} \cap \overline{C}| =$

(b) How many numbers between 1 and 10,000 are not multiples of 4, 10 or 7?

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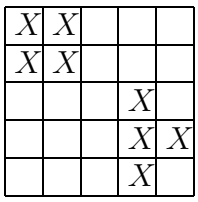
6. (10 points) Find the number of integer solutions to

$$x_1 + x_2 + x_3 + x_4 = 20, \qquad 0 \le x_1 \le 15,\ 0 \le x_2 \le 5,\ 0 \le x_3 \le 5,\ 0 \le x_4 \le 5.$$

7. (10 points) How many ways can you put 5 non-attacking rooks on the 5-by-5 chessboard with forbidden positions shown.



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8. (6 points) Let h_n denote the number of ways in which the squares of a 1-by-n chessboard can be colored red, green, yellow or purple so that no two adjoining squares are colored purple. Find a recurrence relation satisfied by h_n.

9. (8 points) Give the generating function $g(x) = \sum_{n=0}^{\infty} h_n x^n$ for the recurrence relation

$$h_n = 2h_{n-1} - 3h_{n-2}, \quad h_0 = 3, h_1 = 5.$$

10. (12 points) Let h_n denote the number of ways to choose n pieces of fruit from a choice of (identical and unlimited) apples, oranges and mangoes, where the oranges come in packs of three and you want at most two mangoes.

(a) Give the generating function $g(x) = \sum_{n=0}^{\infty} h_n x^n$.

(b) Simplify $g(x)$.

(c) Obtain a formula for h_n.

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