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MATH 510 Discrete Math – **Exam I**

Wednesday, October 1, 2003

Check that that you have all three pages - page two is on the back of page one.
Show all your work and reasoning. If you have sufficient time evaluate factorials and binomial coefficients.
If you apply the box principle make sure you indicate clearly what boxes you are using.

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1. (15 points) How many 5 character passwords can be made using only $c, x, y, z, 1, 3, 5, 7$ or 9?

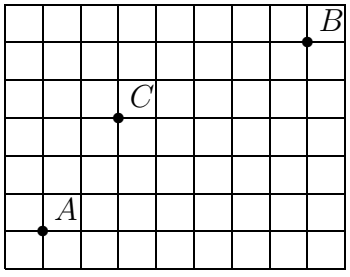
(a) Assuming that repeats are allowed. e.g. $c37x7$, $737xc$, $1111z$, etc.

(b) Assuming that no character is repeated. e.g. $c15y7$, $971xc$, $cx179$, etc.

(c) How many of the passwords in (a) contain two letters and three digits? e.g. $7c3x3$, $113xx$, $37y5x$, etc.

2. (10 points) The streets of a city run North-South and East-West. Assume in the following that you always take the shortest route.

(a) How many routes are there from A to B?



(b) How many routes from A to B pass through C?

3. (5 points) Count the permutations of the letters of the word AARDVARKS.

4. (5 points) A committee of 3 and a committee of 4 are to be chosen from a pool of 10 people. How many different ways can the committees be picked, assuming that no one has to serve on both committees.

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5. (10 points) Britney arranges her 7 pets (3 hamsters, 3 rabbits and a guinea-pig) in a ring. Assume that her pets are all different colors and easily distinguishable.

(a) How many circular arrangements are possible?

(b) How many of these arrangements do not have two rabbits or two hamsters together?

6. (10 points) (a) How many ways can 6 non-attacking rooks be placed on a 6×6 chess-board?

(b) What if 4 of the rooks are red and 2 blue?

7. (10 points) How many ways can you choose 10 pieces of fruit from a choice of apples, oranges, bananas and pears under the following assumptions. Assume fruit of the same type is indistinguishable.

(a) At least 10 fruits of each type are available.

(b) At least 10 apples, oranges and pears are available but only 3 bananas.

8. (10 points) Count the number of integer solutions to

$$x_1 + x_2 + x_3 + x_4 = 12$$

with

(a) $x_1, x_2, x_3, x_4 \geq 0$

(b) $x_1 \geq 3$, $x_2 \geq 0$, $x_3 \geq -1$, $x_4 \geq 0$.

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9. (10 points) Bob chooses 11 (distinct) numbers from 1,2,3,4,...,19.
(a) Prove that he has chosen two numbers whose sum is 20.

(b) Prove that he has also chosen two integers a, b with a dividing b.

10. (5 points) A small class-room has two rows of six seats. There are 9 students, 3 of whom always sit in the front row, 4 always sit in the back row and the remaining 2 can sit anywhere. How many ways can the students be seated?

11. (5 points) What is the coefficient of x^4y^3 in the expansion of $(2x-3y)^7$?

12. (10 points) State the binomial theorem for

$$(1+x)^n =$$

and use it to evaluate the following sums:

(a) $\displaystyle\sum_{j=0}^{n} \binom{n}{j} 5^j$

(b) $\displaystyle\sum_{j=0}^{n} j\binom{n}{j} 5^j$

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