

5. (6 points) Use Fermat factorization to factor 1890353 into two primes.

6. (10 points) (a) $\sigma(700) =$

(b) If $F(n) = \sum_{d|n} \mu(d)\phi(d)$ then $F(700) =$

7. (10 points) (a) Suppose that $d = \text{ord}_m a$. Prove that if $a^n \equiv 1 \pmod{m}$ then $d|n$.

(b) Find (with justification) the order of $\text{ord}_m b$ if $b^8 \equiv -1 \pmod{m}$ with $m \geq 2$.

8. (6 points) Suppose that $g(n)$ is a multiplicative function satisfying $\tau^2(n) = \sum_{d|n} g(d)$.

(a) From the Möbius Inversion Formula $g(n) = \sum_{d|n} \text{_____}$.

(b) Evaluate $g(700) =$

9. (9 points) Use the Chinese Remainder Theorem to solve the simultaneous congruences

$$x \equiv -1 \pmod{7}$$

$$x \equiv 3 \pmod{15}$$

$$x \equiv 5 \pmod{8}$$

10. (8 points) (a) For which distinct odd primes p, q does the Legendre symbol satisfy $\left(\frac{p}{q}\right) = -\left(\frac{q}{p}\right)$?

(b) Evaluate the Legendre symbol $\left(\frac{431}{1097}\right)$

11. (22 points) Circle True (T) or False (F).

T F (a) If $a^{n-1} \equiv 1 \pmod{n}$ then n is prime.

T F (b) $2^{88} + 1$ is a factor of $2^{880} + 1$.

T F (c) $2^3 \parallel 23456qw78xyz45616$

T F (d) $5^{42} \parallel 211!$

T F (e) If $\{a, b, c, d, e\}$ is a complete system of residues $\pmod{5}$ then so is $\{0, b-a, c-a, d-a, e-a\}$.

T F (f) The system $x \equiv 7 \pmod{12}$ and $x \equiv 11 \pmod{20}$ has more than one solution $\pmod{240}$.

T F (g) The Fibonacci numbers satisfy $f_{n+3} = 2f_{n+1} + f_n$.

T F (h) $\underbrace{111112111112111112111112111112111112111112111112}_{6 \text{ times}} \equiv 7 \pmod{13}$.

T F (i) If $2^5 \parallel a$ and $2 \parallel b$ then $2^{17} \parallel a^3 + b^2$.

T F (j) $294409 = 37 \cdot 73 \cdot 109$ is a Carmichael number.

T F (k) If $f(n)$ is totally multiplicative then $F(n) = \sum_{d|n} f(d)$ is totally multiplicative.

12. (10 points) Find the continued fraction expansion of $\sqrt{14}$.

(b) Find the quadratic α with continued fraction expansion $\alpha = [2, 1]$.

13. (12 points) (a) Use the Euclidean algorithm to compute the greatest common divisor (2167,1727)

(b) Solve the linear equation $2167x - 1727y = 143$ or explain why there are no solutions.

(c) Solve the linear congruence $1727x \equiv 132 \pmod{2167}$ or say why no solutions exist.

14. (9 points) A binary number $x_1x_2x_3x_4x_5$ has been encoded as 866 using the knapsack cipher sequence (306, 374, 233, 19, 259) obtained from a super-increasing sequence using modulus 464 and multiplier 17. Recover the super-increasing sequence and decipher the original binary number.

15. (12 points) (a) Calculate the continued fraction expansion of 4.54609

(b) Calculate the continued fraction convergents

(c) A juice company wants its storage tanks to contain a whole number of liters and roughly a whole number of imperial gallons where 1 imperial gallon=4.54609 liters. What would be sensible choices of tank size if the tanks should contain between 1500 and 2500 gallons?